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Automation and Remote Control

(The Soviet Journal *Avtomatika i Telemekhanika* in English Translation)

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Automation and Remote Control

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RESULTS OF THE FIRST CONGRESS OF THE INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL (IFAC)

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 9, pp. 1229-1239, September, 1960

The sessions of the IFAC First Congress, which was prepared and organized by the National Committee of the Soviet Union for Automatic Control, were held from June 27 to July 2 on the premises of the Lomonosov Moscow State University. The Congress was a very important event in the history of automatic-control development. Its import can be judged by the following figures. The number of participants at the Congress included 1190 delegates from 29 countries, and almost as many guests. During the sessions of the 21 Congress sections, 270 reports were heard and discussed; over 400 statements were pronounced in the discussions of these reports.

The Congress was opened by the address of the IFAC president, Prof. A. M. Letov, on June 27 at the first plenum. The IFAC past-president, G. Chestnut, welcomed the Congress delegates in his speech at this session. The first Vice-President of the Council of Ministers of the USSR, Comrade A. N. Kosygin addressed the Congress delegates in the name of the Soviet Government. The Congress delegates were greeted by the President of the Academy of Sciences USSR, Academician A. N. Nesmeyanov, Prof. E. Gerecke (Switzerland), Prof. R. Oldenburger (USA), and Comrade Wu Juyen (Chinese People's Republic).

In his closing address at the first plenum, the president of the National Committee of the Soviet Union for Automatic Control, Academician V. A. Trapeznikov, read the report "Automation and Mankind."

The subsequent work of the Congress took place during the sessions of different sections. The problems in automatic-control theory were considered in nine sections; five sections were devoted to problems in providing the technological basis of automation; the work in six sections concerned the problems in applying automation theory and technology in automating industrial processes; one section was devoted to general problems pertaining to automatic control as a scientificotechnical branch of science.

The closing plenum was held on June 2. During this session, the following persons surveyed the scientificotechnical problems discussed at the Congress: the president of the IFAC Scientificotechnical Committee on Theory, Academician B. N. Petrov (USSR), the president of the IFAC Scientificotechnical Committee on Automation Technology, Prof. G. Boromiss (Hungary), and the president of the IFAC Scientificotechnical Committee on Practical Application, Prof. D. Mosley (USA). The president of the IFAC Consulting Committee, Prof. D, Eckman (USA), made an announcement concerning the activities of this committee. The president of the IFAC Scientificotechnical Committee on Terminology, Prof. E. Gerecke (Switzerland), dwelled on the problems in the work of this committee. The Congress was closed by the final address of the IFAC president, Prof. A. M. Letov.

SURVEY OF THE SCIENTIFICOTECHNICAL PROBLEMS CONSIDERED AT THE CONGRESS*

Automatic-Control Theory

The theory of automatic control was treated in 139 reports. The reports themselves as well as the ensuing discussions were a vivid demonstration of the high level attained in the development of the theory of automatic control, which is based on a widespread use of rigorous mathematical methods and is concerned with the solution of basic problems in the analysis and synthesis of modern automation systems.

Theoretical problems were discussed in the work of nine sections.

Twenty-three reports were discussed in the Section dealing with the Theory of Continuous Linear Systems. Reports devoted to problems in the invariance theory and the theory of multiple feedback and autonomous control systems created the greatest interest and the liveliest discussions. The discussion of these problems showed that, at the present time, the theory provides the basic principles for the design of automation sys-

*This survey was compiled from the above-mentioned reviewing reports by the three presidents of the IFAC Scientificotechnical Committees: Academician B. N. Petrov (USSR, Committee on Theory), Prof. G. Boromiss (Hungary, Committee on Technology), and Prof. D. Mosley (USA, Committee on Practical Application). This survey does not review the work of the Section for General Problems in Automatic Control (terminology, training of specialists, etc.).

tems which respond little to undesirable disturbances and accurately follow the control inputs.

In the work of this Section, much attention was paid to the analysis of multiple feedback control systems and general methods for their synthesis, methods for improving the dynamic properties of control systems with one and several controlled parameters, and also problems in the theory of systems with lag elements.

A number of reports were devoted to improved and generalized methods for determining the stability and the performance of automatic-control systems.

Fourteen reports were discussed in the work of the Section dealing with the Theory of Continuous Nonlinear Systems. Exact as well as approximate methods for the investigation of such systems were surveyed.

A number of reports were concerned with the development of the describing-function method. It was shown that this method, which gained widespread use in engineering practice, can be extended, in matrix representation, to the case of multiperiod oscillations, subharmonic and superharmonic resonance phenomena, oscillation synchronization, and parametric amplification. An approach to the solution of some problems in the synthesis of nonlinear systems subject to random disturbances, based on a combination of the describing-function and the statistical-linearization methods, was indicated.

It was also shown that, by using different variants of the describing-function method, it is possible to study the effect of the limited power of feed sources in control systems and to find conditions for suppressing self-oscillations by external periodic inputs, and to determine the conditions for suppressing self-oscillations in systems with backlash. One of the reports presented a variant of the describing-function method where Nichols loci are used.

Two points of view were expressed in the discussion on the application of the describing-function method. One of them states that, regardless of the extreme effectiveness of the describing-function method, it should be used with a certain caution, since the fact that no self-oscillations are detected by means of this method does not confirm the absence of any complex periodic motions in a system. According to the other point of view, the simple and efficient describing-function method yields accurate results in practically all problems encountered in practice, and can be developed so that it yields subharmonic solutions.

Besides the above problems, the following problems were considered in this Section: effect of programmed variation of the regulator gain on the control performance, problems in calculating the static characteristics of complex nonlinear systems, and a new method for calculating the integral quadratic criterion of nonlinear systems.

In the Section dealing with the Theory of Discrete Systems, twenty-five reports were discussed; problems in the theory of relay (off-on), sampled-data, and digital automation systems were considered. Along with problems in the analysis and synthesis of discrete systems intended for the solution of ordinary control problems, much attention was paid to problems in designing optimum and self-adjusting discrete systems.

The reports devoted to the theory of relay systems presented exact and approximate methods for the analysis of subharmonic oscillations, as well as methods for the design of relay systems with optimum quickness of response which are subject to random disturbances.

Reports relating to the theory of sampled-data systems were the most numerous in this Section. A survey of mathematical methods for the investigation of such systems was presented; among the problems discussed were those encountered in the design of multimode and multiple feedback sampled-data systems, problems in the design of sampled-data systems with variable parameters, methods for statistical calculation of sampled-data systems, and methods for investigating nonlinear sampled-data systems. The problems in the design of optimum and self-adjusting sampled-data systems whose structure comprises feedback with respect to the performance criterion were also considered. A number of reports were devoted to the theory of relay sampled-data (digital) systems. The simplest systems of this type were analyzed by using the phase-plane method and by introducing lags with random variation. The recently developed branches of the theory of digital automatic systems, including the determination of the effect of quantization with respect to a certain level, methods for determining periodic modes, the synthesis of systems with lag compensation, and the synthesis of optimum systems, were presented. The liveliest discussion developed with regard to problems in the application of digital computers in sampled-data control systems and the application of the phase-space and operator methods in investigating sampled-data systems. The necessity for developing methods for the analysis and synthesis of nonlinear sampled-data systems and sampled-data systems subject to random disturbances was emphasized in the discussion.

Eighteen reports were discussed in the work of the Section dealing with Stochastic Problems. One of the main problems in this field is that of the statistical analysis and synthesis of nonlinear systems. Three different trends in work on this group of statistical automation problems became apparent through the reports and the ensuing discussions. The first trend pertains to the category of problems which include the development and generalization of the statistical-linearization method and its application, as well as the formulation of an exact theory for a given class of nonlinear systems, which would be based on the theory of Markov's random processes, and the development of methods for the synthesis of systems which can be reduced to linear systems. During the discussions great interest on the part of scientists from different countries was shown in the statistical-linearization method,

which is conceptually close to the harmonic linearization method and the deterministic theory of nonlinear systems. In addition to what was said in the course of the discussions regarding the application of the Markov theory of random processes in the analysis of automatic systems, Soviet scientists reported a number of interesting results which were not known outside of our country before. In the discussion concerning problems in the statistical theory of nonlinear systems capable of reduction to linear systems, a number of interesting results, promising successful application in the design of self-adjusting and self-optimizing systems, were announced.

The second trend pertains to problems in the theory of statistical synthesis of linear systems. The discussion revealed the appearance in this field of two new competitive approaches in the USA which have been well developed in the papers of American as well as Soviet scientists. These two approaches are the application of the dynamic programming method for the determination of the optimum program for the controlling section of a system and the development of methods for the direct determination of differential or difference equations of optimum systems.

The third trend pertains to problems in finding the algorithms of optimum systems. It was demonstrated in the discussion that the majority of problems in detecting and processing information in the presence of noise and random disturbances can be considered from a unique point of view and solved by a unique theoretical method.

In addition to the reports and the discussion concerning the above three basic trends, the work in this Section also included the discussion of problems in experimental determination of characteristics of control systems and systems to be controlled,as well as of statistical characteristics of random processes, problems in statistical calculation of linear systems, problems in determining errors in the conversion of continuous information into discrete information, as well as problems in applying the information-theory methods to the determination of the transmission capacity of automatic systems and their components.

Twelve reports were discussed in the Section dealing with the Theory of Optimum Systems; general methods for investigating optimization problems,as well as the results obtained in solving individual problems,were considered. The general methods for the investigation of optimum systems are mainly connected with the development of methods for the solution of nonclassical variation problems. The report dealing with these problems was devoted to the maximum principle and the theory of optimum automatic-control systems and aroused great interest and a lively discussion. The participants in the discussion remarked that this principle provides a powerful tool for the investigation of nonlinear optimization systems, while the restrictions imposed on the control functions are of a general nature; at present, it is one of the most general principles, thus making it possible to solve a large number of problems by means of it.

Much attention was paid to problems met in the synthesis of optimum systems. The inverse time methods and methods for construction of isosurfaces in the system phase space, both of which are based on the application of the maximum principle, were discussed. Practical examples of system synthesis performed by the above-mentioned methods were given.

The connection between the maximum principle and the dynamic programming method was analyzed during the discussion. It was noted that the dynamic programming method, when applied to optimum control problems, yields functional equations which are closely related to the maximum principle equations. However, it was indicated that the application of the dynamic programming method involves considerable calculation difficulties.

Great interest was exhibited in the formulation and solution of some problems in optimum control of systems with distributed parameters (which are described by partial differential equations),as well as of general and particular problems in the theory of optimum linear-control systems.

Twenty reports were presented to the Section dealing with the Theory of Self-Adjusting Systems. The items discussed in this Section were biological and mechanical systems capable of adaptation to changing conditions,with the aim of securing such an interaction between the system to be controlled and the surrounding medium, for which, in a given sense, the optimum state of the system to be controlled is secured. Systems where adaptation is secured by automatic scanning as well as by changing the system state with respect to an open-loop or closed-loop cycle (without automatic scanning) were considered.

Problems in the following branches of the theory of automatic control were considered: the statistical theory of automatic optimization systems, dynamics of self-adjusting systems, bioelectric control, the study of characteristics of physiological control systems, and the theory of self-adjusting autopilots.

Interesting reports were presented on the theory and application of computers for the approximate determination of characteristics of systems subject to automatic and self-adjusting control.

The reports concerning problems in biological cybernetics brought about an especially lively discussion, which showed that Soviet, Yugoslav, and American scientists contributed much to the development of principles for the design of systems in many ways similar to living organisms. Six reports were devoted to the results obtained in studying biological self-adapting systems. The results obtained by a group of scientists in studying problems of bioelectric control attracted deserved atten-

tion; these results make it possible to use biocurrents in a living organism for the control of artificial organs supplanting the lost ones.

At present, the methods and procedures used in automatic-control theory are applied in the investigation of biological systems to an ever-increasing extent. This trend in cybernetics, which is beginning to yield interesting results, was reflected in the reports and in the ensuing discussions. It should be noted that the discussion in the work of this Section was perhaps the liveliest, indicating the especially great interest of the scientific world in this important and promising direction of the automatic-control theory.

It was indicated in a number of reports and in the following discussions that it was necessary to investigate and develop self-adjusting systems capable of operating under conditions of rapid drift of characteristics and parameters of the systems to be controlled. These problems are closely connected with problems in the stability of the self-adjustment process, and, therefore, the importance of exact mathematical formulation of stability conditions in self-adjusting systems was repeatedly emphasized in the discussions.

Seven reports were discussed in the Section dealing with the Theory of Structures and Signal Flow Diagrams. A number of important problems in the theory of relay devices and the theory of signal flow diagrams were discussed; these problems included: development of methods for the synthesis of relay devices and problems in the design of relay structures with assigned reliability; problems in the application of a new logic language for the description of discrete time sequences; problems in the mechanization of the analysis and synthesis of relay devices; problems in noise stability and the efficiency of information transfer in remote measurement systems. However, due to the fact that no reports from a number of eminent scientists from abroad working in this field, scientists from the USA in particular, were submitted to the Section, many other important problems in the modern theory of structures and the theory of signal flow diagrams were not discussed. This gap was partially filled by scientists from the Soviet Union and abroad, who took part in the discussion.

Considering that the structural theory of relay devices is one of the important branches of the modern automatic-control theory, it was planned to hold in the nearest future a special symposium on the theory of relay devices and related scientific fields.

The discussion in the Section dealing with Special Mathematical Problems comprised thirteen reports pertaining to three special mathematical problems in automatic-control theory.

The first problem pertains to the determination and investigation of steady-state conditions in automatic control systems which are described by differential equations with continuous right-hand sides.

A number of important aspects of this problem were treated in the reports: a more rigorous and exact mathematical definition of the steady-state conditions concept was given; the results obtained in investigating the dependence of the periodic solutions of a system of differential equations on its parameters were described.

The second problem pertains to the investigation of nonlinear systems with discontinuous characteristics. With regard to these systems, problems in determining steady-state conditions were discussed.

The third problem pertains to the stability of automatic-control systems. The main problems treated in the reports, and in the lively discussion they gave rise to, pertain to the further development of methods of determining stability by using the second Lyapunov method.

Eleven reports were discussed in the Section dealing with Simulation and Experimental Investigation Methods. The application of simulation methods, where not only analog, but also digital and analog-digital (combined) devices are used for the investigation and design of complex control systems, is a separate and important problem.

The solution of this group of problems will help in mechanizing certain human mental processes and considerably increase their efficiency. By using simulation methods, the experimental techniques are rendered more accurate and prerequisites for a more efficient use of equipment for the testing of actual systems are secured. The development of equipment and methods for digital simulation was widely reflected in the reports and the ensuing discussion; with regard to programmed-control systems, new methods for the reproduction of second-order curves by means of digital computers were proposed. The application of combined digital-analog computers (in particular, for the reproduction of nuclear reactor kinetics) was also considered.

It was noted that, besides improving the accuracy, the application of digital-simulation methods considerably increases the maximum allowable range in which the variables change.

A method which was proposed for the solution of partial differential equations by means of electronic models aroused great interest; this method is based on the connection between the equations to be solved and the processes which appear in practical circuits subject to random disturbances.

One of the reports mentioned the development of an original simulating device which operates with high-frequency alternating current and which consists of passive circuits combined with follow-up multiplier systems; this device has a relatively high accuracy and is reversible, which considerably enlarges its scope.

It was noted in the discussion that the new ideas in the application of electronic models for the optimiz-

ation of the performance of systems used for the automatic control of industrial processes offer great opportunities for the design of optimum systems, as well as for obtaining more accurate information on the systems to be controlled.

Automation and Remote-Control Technology

Fifty-five of the reports submitted to the Congress were devoted to problems in construction, investigation, and design of automation and remote-control equipment. These reports were discussed at the sessions of five Sections.

Nine reports were discussed in the Section dealing with Electric and Magnetic Components of Control Systems. Problems in the theory of magnetic amplifiers (three reports) were most thoroughly treated. A detailed theoretical analysis of the effect of dynamic properties of magnetic cores in the presence of eddy currents and magnetic viscosity on the characteristics of the basic magnetic amplifier types was presented for the first time in one of these reports. These results make it possible to determine the theoretical characteristics of different amplifier types and to find ways for their improvement.

Moreover, among the problems considered in the work of this Section were problems in the design of logic elements and their use in control circuits; problems pertaining to measurement methods; and problems in the design and investigation of some special electric automation devices (photoresistors and quick-action step motors).

In the report devoted to logic components, attention was drawn to the fact that a system of logic elements can be without a special memory component, since, in the design of circuits requiring the application of such a component, one can use standard logic components and circuit feedbacks.

Ten reports were discussed in the Section dealing with Electrical Computers and Simulating Devices and Programming Machines and Components. According to their character, the problems considered in these reports can be divided into three groups.

The first group was dealt with in reports devoted to problems in the design of new computers. The following items were considered in these reports: the design of a new computer for algebraic functions of a complex variable; the design of a synthesizer in the shape of a nonuniform electronic lag line; the design of a discretely acting computer of the electric-grid type intended for the solution of unsteady-state field equations. Also among the items discussed was a system for the control of industrial processes containing a correcting computer having a remote measurement selector for centralized control. (A ternary code is used in the selector of this computer.) Finally, this group included a report on the application of automatic optimizers for automatic synthesis and the solution of variation problems.

The reports of the second group related to computers for the automation of industrial processes. The SKZhAT electronic digital computer, designed for the control of such processes as chemical synthesis, distillation, and blast-furnace processes, belongs to this class of computers.

The reports of the third group treated problems in the improvement of computer components. There was a communication on the automatic zero stabilization in operating amplifiers, for which a number of circuits for reducing the zero drift were proposed and a comparative evaluation of different circuits was given. The problem of the design of nonlinear computer units, based on semiconductor resistors made of silicon carbide, was discussed. A report devoted to the logical synthesis of functional generators was read.

The reports and the ensuing discussion showed the great importance of computer techniques in the solution of modern problems in automatic control.

Eight reports were discussed in the Section dealing with Data Transmitters, Components, and Automation and Remote-Control Systems for Industrial Processes. The main trends in the development of remote control and measurement techniques were discussed in the reports presented to this Section. In the field of remote measurement, the main development trend consists in the application of contactless digital and code-pulse systems; in the field of remote control of a large number of distributed systems, this trend can also be reduced to the application of contactless equipment. In the work of this Section, much attention was paid to problems in the operational accuracy and reliability of remote measurement and control systems. Among the items discussed were performance criteria for remote control systems based on different characteristics, such as the specific signal content index, the information transfer quality index, the information channel utilization factor, and the technical efficiency.

Ten reports were discussed in the Section dealing with Pneumatic Devices in Automation and Computer Techniques. Problems in the design of the following pneumatic and hydraulic automation components were considered: pneumatic throttles of the nozzle-flapper type which control devices having jet pipes, servovalves with discharge feedback, and long lines which connect the control valve to the drive. Moreover, there was a report on the general principles of the theory and design of pneumatic automation elements. This generalized report aroused great interest and a lively discussion.

Among the reports presented to this Section, the most interesting were those devoted to the development of pneumatic equipment designed for mathematical and logic operations. It was shown that pneumatic devices can be used successfully for the realization of analogous

simulating devices and that the most diverse relay circuits for algebraic and time logic functions can be constructed.

A report containing an analysis of the dynamics of the copying machine hydraulic drive and a report devoted to the investigation of automatic-control systems for complex hydrodynamic machines aroused great interest. A report considering problems of the dynamic suspension of machine parts in a viscous liquid brought about a lively discussion.

Eighteen reports, divisible into three groups, were discussed in the Section dealing with Automatic-Control Equipment and Devices.

Sensing elements were considered in the reports of the first group. The results obtained in the development of a thermomagnetic compensation gas analyzer to be used for the accurate measurement of high oxygen concentrations were discussed. The application of radioactive isotopes in automatic-control systems for mining extraction and sinking machines was demonstrated.

The reports of the second group were devoted to improved measurement methods.

Methods for the automation of sampling and chemical analysis and for the automation of measurements in plotting the dynamic characteristics of nuclear reactors were considered. A report presenting a method for determination of the hunting angle of the earth's satellites by means of a gyroscope and by means of devices using nuclear radiation was discussed.

The third group of reports was devoted to the results achieved in the design of individual intrument parts. These achievements include, for instance, the production of components with a higher temperature stability and a method for determining optimum vibrations, which exclude the effect of Coulomb friction.

Application of Theory and Technology for the Automation of Industrial Processes

Problems in practical application of the automation theory and technology were treated in sixty-seven reports. These reports were discussed at the sessions of six Sections.

Ten reports were discussed in the Section concerned with Automation of Machine Construction. In the work of this Section, most attention was paid to problems in the design and practical realization of automatic-control systems for different machines. These problems comprised the selection of the simplest and most efficient devices for recording different types of information; the selection of computer and programming devices; the determination of the optimum operating conditions for a given machine. The groups of problems considered in the work of this Section encompassed the main trends in the development of programmed control techniques for machine tools, as well as a number of special problems connected with the operational reliability of machine tools.

In the discussion following the reading of reports, it was noted that digital-control methods play an especially important role in machine construction.

Twenty-two reports were discussed in the Section occupied with the Automation of Power Systems.

An analysis of the reports submitted to this Section indicates that a number of new and important scientific and practical results have been achieved. The most important are the following: Equations which determine the main characteristics of the most important power-generating systems to be controlled (boiler units, nuclear reactors, and heat exchangers) have been derived. Experimental data characterizing the properties of typical systems to be controlled (industrial furnaces, power-generating devices, etc.) have been obtained. New criteria for the optimum operating conditions of systems to be controlled, which take into account the effect of the controlled variable deviations and of changes in the energy carrier flow on the efficiency of devices, have been proposed. A number of original schemes for the automation of the most important systems to be controlled (boiler units, nuclear reactors, metallurgical furnaces, and fractionating columns) have been described. New views have been expressed on the principles of operating mode optimization to be applied to controlled systems, and first attempts have been made to determine the scope of different types of self-adjusting systems. Specifications for the characteristics and parameters of certain systems to be controlled, based on control conditions, have been formulated and substantiated.

Nevertheless, the discussion showed that, at the present time, no substantial progress toward solving the following important problems in industrial automation has been achieved: general principles for the design of automatic-control systems for complex industrial combines have not yet been formulated, and methods for improving the operational reliability of automatic-control systems have yet to be devised.

Eight reports were presented to the Section concerned with Automation in the Chemical and Oil-Refining Industries. The main attention in the work of this Section was centered on two problems. The first pertained to the determination of dynamic properties of systems to be controlled in the chemical industry, and the second problem involved the application of computers for the analysis, calculation, and control of technological processes. The considerable number of reports devoted to the study of dynamics of systems to be controlled and the animated discussion called forth by them convincingly prove that the development of computer techniques does not lessen the importance of determining the dynamic properties of systems to be controlled in analytical form. The study of characteristics of systems to be controlled still remains one of the most important stages in designing complex automation systems.

However, the problems encountered in the application of computers to automatic-control systems for technological processes were considered only in their most general form, indicating that computer techniques have not yet been introduced in industrial automation on a large scale.

In the reports and the discussion, attention was drawn to the necessity of directing efforts toward the solution of the following important problems: determination of dynamic properties of systems to be controlled and of entire technological chains; development of optimization algorithms for technological processes; establishment of efficiency criteria for a justified application of computers and the development of general principles of complex automation.

Ten reports were discussed in the Section dealing with Automated Electric Drives and Electrical Machines. The greatest interest was aroused by problems met in seeking new methods for accelerating transient processes in automated electric drive systems, problems in the design of new components and devices, and methods for the synthesis of nonlinear automated drive systems.

The reports and the discussion emphasized the necessity for a further development and a more widespread application of methods for the analysis and synthesis of automated drive systems with optimum quickness of response, the necessity for a more widespread application of computers in systems for the control of electrical drives, the need for devising new compact and reliable automation components and devices, and the necessity of developing improved measuring instruments of high accuracy.

Thirteen reports were discussed in the Section occupied with Automation of Metallurgical Processes. Among the problems discussed were the determination of static and dynamic characteristics of systems to be controlled, problems met in the design of new automatic control systems for complex metallurgical units, and problems in the design of new, improved components for automation systems. Equations were adduced which fairly accurately described the static and dynamic properties of a number of important metallurgical devices (rolling mills, boiling-layer furnaces, heating wells, etc.); a large number of pertinent experimental data also were given.

It was announced that new control systems, based on self-adjustment principles and the extensive use of computer techniques, were developed and tested in industry. Methods for the complex automation of individual units and of entire shops and plants, where process optimization requirements, determined by a number of technological and efficiency specifications are taken into account, were outlined in these papers. Information was provided on the development of new original and efficient components in automatic-control systems for industrial processes (electron-optical equipment for the measurement of dimensions and rate of rollings, pneumatic meters with vane-shaped profiles, thermoelements, etc.).

The discussion that developed in the work of this Section showed that, for the further development of automation in metallurgy, it will be necessary to devise methods for the mathematical description of characteristics of systems to be controlled, to develop unique principles for the design of automatic-control systems, and to establish performance criteria for automatic-control systems to be used in complex metallurgical devices. It was also indicated that methods of automatic control with respect to disturbances, statistical methods for determining disturbances acting on a system, and self-adjusting systems for devices whose operating mode is optimized by means of models should be developed and utilized on a larger scale; that it is necessary to further improve and increase the reliability of components in automatic-control systems for metallurgical devices, which is especially pertinent to data transmitters, final mechanisms and regulating components, since they operate under heavy-duty conditions in metallurgical plants.

Four reports were discussed in the Section dealing with Unclassified Problems. The greatest interest was shown in reports devoted to problems in control and regulation in the cement and sugar industries. It was remarked in the discussion that these reports contain many valuable results obtained in the field of automation in sugar refining and cement production; however, it was also noted that the complex processes occurring in blast furnaces have not been thoroughly analyzed in developing automatic-control systems to be used in the cement industry.

In conclusion, it should be noted that the numerous discussions which developed during the work of the Congress were entirely businesslike, that they contained a great deal of well-meant criticism, and that, consequently, they were fruitful.

The work of the Congress, the slogan of which was "The theory should be widely applicable, the technology should have the maximum reliability, and the automation systems should be highly efficient," showed that this slogan very accurately characterizes the modern trends in automatic-control development.

DUAL-CONTROL THEORY. I

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Some fundamental problems in communication theory and control theory are compared. The problem of designing an optimum (in the statistical sense) closed-loop dual-control system, is formulated. Its solution, as well as examples and some generalizations, will be given in parts II, III, and IV.

Introduction

A general block diagram of signal transmission, as investigated in communication theory is shown in Fig. 1. The transmitted signal x^* proceeds from the transmitting device A to the communication channel H^*. The mixing of signal and interference (noise) h^* now takes place. The resultant signal y^* represents a mixture of the transmitted signal and the interference. The resultant signal proceeds to the input of receiver B. The optimum receiver problem consists in obtaining the signal $\underline{x}$ at its output, such that it is, in a specified sense, closest to the transmitted signal x^* or to some transformation of the signal x^*. The mathematical side of the problems related to such systems has been the subject of important investigations by A. N. Kolmogorov [1], N. Wiener [2], C. E. Shannon [3], and A. Wald [4]. This type of system was investigated in the works on communication theory by V. A. Kotel'nikov [5], D. Middleton, D. Van Meter [6], and others. The cited works differ in their various approaches to the problem, but are all basically concerned with the investigation of the scheme represented by the block diagram in Fig. 1. The results obtained in the above-cited works, and in particular the Kolmogorov-Wiener theory, have proved useful in formulating the statistical theory of automatic-control systems. This theory has been expounded in the books of V. S. Pugachev [7], J. H. Laning, Jr., and R. H. Battin [8], and others. The fullest consideration has been given to the theory of linear systems. If a system is linear, then whatever the closed-loop system, it is easy to obtain an open-loop system equivalent to it. That is why the automatic-control systems, which, as a rule, are closed-loop systems, enable one to use the scheme as in Fig. 1, provided that the system is linear. Some complications and difficulties arise when the interference does not appear at the input of the system, but at the input of the controlled object – the latter being inside the closed-loop network. This also creates difficulties which are not, however, of a fundamental nature. More serious difficulties arise due to the bounds to which the power of the system's signals are subjected. The problem becomes more involved when the controlled object is nonlinear or it is required that an optimum control system, which often proves to be nonlinear, is designed. It is not always possible to proceed from an open nonlinear system to an equivalent closed-loop one; furthermore, this is an extremely involved process. In such a case, the open-loop scheme depicted in Fig. 1 cannot be used in practice. A number of attempts have been made to reduce approximately nonlinear systems to equivalent linear ones (see, for example, the paper of I. E. Kazakov [9]). Such studies are of considerable practical value but do not, in their present state, provide a means of estimating how close the obtained approximation is to the true solution; neither do they enable one to synthesize the optimum system.

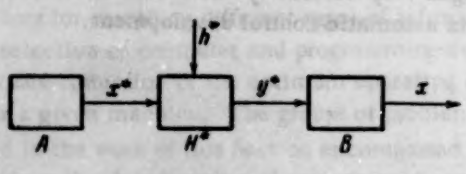

Fig. 1.

In order to be able to solve optimum problems of the control theory, a fundamentally different approach is required. Firstly, a different block diagram replacing that depicted in Fig. 1 is needed. Before selecting a common scheme to be used in automatic-control theory, it seems advisable to have a preliminary survey of certain basic concepts of the theory.

Figure 2, a shows the controlled object B with $\underline{x}$ as its output, $\underline{u}$ as the controller, and $\underline{z}$ as the disturbance (interference). When the system has several inputs and outputs, one can regard $\underline{x}$, $\underline{u}$, and $\underline{z}$ as vector quantities.

The output $\underline{x}$ depends on $\underline{u}$ and $\underline{z}$. This dependence can be described either by a linear or a nonlinear operator, and in a particular case of memoryless systems, only by a function. The interference $\underline{z}$ is generally a function

of time. Thus, since a change in the system's characteristics can be considered a particular result of interference (e.g., the parametric effect), then hereafter everything in the system's characteristics that changes with time will be attributed to interference. If, for example, $\underline{x}$ depends on $\underline{u}$ as in

$$\dot{x} = [a_0 + f_0(t)]\,u^2 + [a_1 + f_1(t)]\,u + a_2 + f_2(t), \quad (1)$$

then the vector

$$z = \{z_0, z_1, z_2\},$$
$$z_0 = f_0(t),\ z_1 = f_1(t),\ z_2 = f_2(t) \quad (2)$$

gives the interference or disturbance, and the formula

$$\dot{x} = [a_0 + z_0]\,u^2 + [a_1 + z_1]\,u + a_2 + z_2 \quad (3)$$

represents a particular operator.

If the object has memory and if, for example, its motion can be described by a differential equation of the nth order, then its state is considered as one of the characteristics of the object as described by the value of the vector $\underline{x}$ in the n-dimensional phase space.

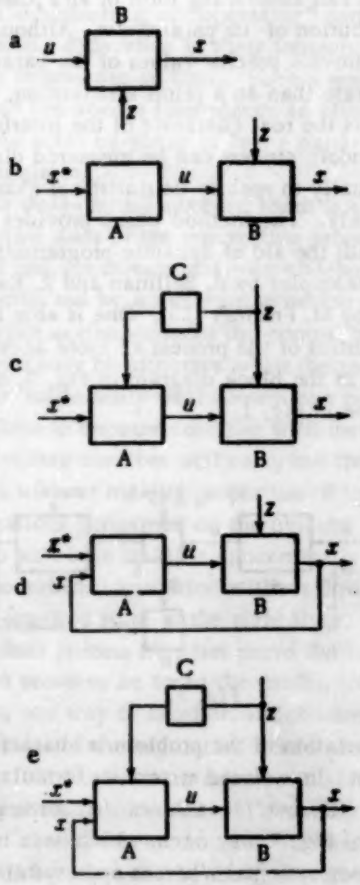

Fig. 2.

Complete information about a system thus consists of information about its operator, interference (noise), and the state of the system. The controlling device may be given and is considered as known. The open-loop systems are automatic-control systems of a simple type. A block diagram of an open-loop system is shown in Fig. 2, b and is of the same character as the one given in Fig. 1. The exciting quantity x^* enters the input of the regulating member A, determining how the output quantity $\underline{x}$ should vary. The output $\underline{u}$ of the regulating device enters the input of the object B under control. The output $\underline{x}$ of the controlled object B does not proceed in this case to the regulating member A.

The required rule of change in $\underline{x}$ can only be implemented when full information about the controlled system is available, i.e., when its operator and state $\underline{x}$ are known, at least at the initial moment of time, as well as the interference $\underline{z}$. The latter should be known a priori at all moments of time, including future moments. The required rule of change in $\underline{x}$ must be one of the admissible ones, such that it can be implemented for the given class of initial states of the system and for a class of controlling motions $\underline{u}$ staying within acceptable bounds.

The above conditions, and, in particular, the a priori full knowledge of the controlled system, cannot be satisfied in practice. This is why an accurate implementation of the required control rule cannot be obtained. Sometimes the interference $\underline{z}$ is not known a priori, but it is possible to measure it with a device C (see Fig. 2, c) and to introduce the outcome of the measurement into the controlling member A. One can then find in the latter the required controlling rule $\underline{u}$. Such a scheme is also an open-loop one. But the scheme depicted in Fig. 2, c differs in some ways from the scheme with complete a priori information about the system, as now future magnitudes of the interference $\underline{z}$ remain unknown. Because of this, the exact implementation of the required rule of variation of the controlling quantity $\underline{x}$ is not always feasible.

When the state $\underline{x}$ of the system is not known, then, generally speaking, it is not possible to implement the required rule of the change in $\underline{x}$. To be able to attain the required variation in $\underline{x}$, or one near to it, a feedback network is needed to feed the output quantity $\underline{x}$ to the input of the controlling member A (see Fig. 2, d). Having compared $\underline{x}$ and x^*, the controlling member generates the regulating action $\underline{u}$, bringing $\underline{x}$ to its required value. The block diagram of Fig. 2, d is a closed-loop scheme, and is of the utmost importance in the automatic-control theory.

A closed-loop network offers far-reaching possibilities not available in an open-loop system. For example, it may be possible for a class of objects B of control to obtain a process x close to the one required even when the interference $\underline{z}$ remains unknown and incapable

of measurement. Let, for example, the interference $\underline{z}$, together with the controlling action $\underline{u}$, be applied at the input of the controlled object B, the latter representing an inertia member. If the quantity $x^* = x$ is to be attained, the regulating member can be implemented in the form of an amplifier of high gain $k \gg 1$, the difference $x^* - x$ being sent to its input.

It is not difficult to see that the requirement $x^* = x$ will be satisfied approximately, whatever the continuously varying interference $\underline{z}$, provided $\underline{k}$ is sufficiently high and the bounds of variability of $\underline{u}$ are such that the interference $\underline{z}$ can be compensated by $\underline{u}$.

This principle of neutralizing the interference can be generalized and applied to cases considerably more involved, combining the system's accuracy with its stability. A detailed analysis of the applicability of this principle was carried out by M. V. Meerov in his monograph [10].

When the interference $\underline{z}$ can be measured, it is possible to implement a combined system (see Fig. 2, e) of measurement of the state of the controlled system $\underline{x}$ as well as of its interference. Such systems are of considerable practical value. We shall not, however, concern ourselves with them but shall limit the study to the "pure" type of closed-loop systems, considering them as being of primary importance.

The input quantity x^* may be previously unknown; usually, neither do we have any prior knowledge of the interference $\underline{z}$. Consequently, these processes become random, and, in a favorable case, the a priori information is limited to our knowledge of their statistical characteristics. Such processes may be regarded as belonging to a class of curves $x^*(\lambda)$ and $z(\mu)$ where λ and μ are parameter vectors $(\lambda_1, \ldots, \lambda_q)$ and $(\mu_1, \ldots, \mu_m)$, respectively, with their probability distributions either known or unknown.

In communication channels connecting the blocks of a system, the errors of measurement or noise can be regarded as subsidiary random processes as well, with either known or unknown characteristics. Thus, the analysis of a control system and the synthesis of the regulating member can be regarded as problems of a statistical nature. The problem should be solved for an over-all block diagram in which all the above features of an automatic-control system are reflected. Such a block diagram is depicted in Fig. 3; it is the subject of the present paper as well as that of further papers in this series.

The input quantity x^* proceeds to the input of the controlling member A through channel H^* where it becomes mixed with noise h^*. Thus, the quantity y^* entering the input of A is generally not equal to the actual value of the input quantity x^*. There also exists a class of systems with the external input x^* altogether absent. Generally speaking, however, it cannot be neglected. A similar mixing takes place of the state $\underline{x}$ of controlled object B and noise $\underline{h}$ in channel H; quantity $\underline{y}$ entering A will not, as a rule, be equal to $\underline{x}$. The regulating action $\underline{u}$ proceeds next from A to controlled object B having previously passed through channel G where it was mixed with noise $\underline{g}$. The quantity $\underline{v}$ proceeding to the controlled object is not, as a rule, equal to $\underline{u}$.

Dual Control

One cannot neutralize, in a general case, the interference $\underline{z}$ by a regulation $\underline{u}$ if the interference $\underline{z}$ is not known. Its direct measurement is not, as a matter of fact, often possible. In such a case, an open-loop system is useless. But the closed system in Fig. 3 shows how $\underline{z}$ can be indirectly determined by measuring the input and the output, in and out of object B, by studying its characteristics. The input of controlling member A enters both the input $\underline{v}$ and the output $\underline{x}$ of the object or, in any case, the quantities $\underline{u}$ and $\underline{y}$ related to $\underline{v}$ and $\underline{x}$. The examination of the quantities $\underline{u}$ and $\underline{y}$ provides information on the characteristics of object B. It should be understood that this information is never complete, as the noises $\underline{g}$ and $\underline{h}$ render an exact measurement of B's characteristics impossible; if the actual form of the object's operator is not known either, a full determination of its characteristics would not be possible even in the absence of noise, unless the determination time is infinitely great. The lack of complete information on the disturbance $\underline{z}$ can assume the form of an a posteriori probability distribution of its parameters. Although the latter does not provide precise values of the parameters, it is more accurate than an a priori distribution, as the former reflects the real character of the interference.

If the random process can be measured directly, one is able eventually to specify its statistical characteristics more accurately. The method which provides such improvement with the aid of dynamic programming was discussed in examples by R. Bellman and R. Kalaba [11, 12] and also by M. Freimer [13]. One is able to find the characteristics of the process x^* more accurately in the open part of the block diagram in Fig. 3, or in a similar scheme in Fig. 1.

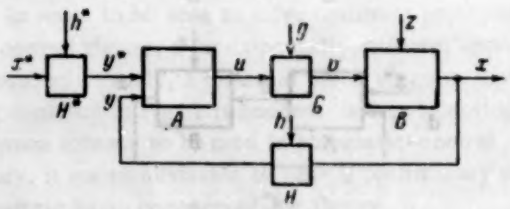

Fig. 3.

This formulation of the problem is characteristic for an open system. In a closed system its formulation becomes totally different. It is shown that some processes in the system of Fig. 3 may occur which have no counterpart in open-loop systems. Whereas open systems can only be studied by passive observations, the study may de-

velop into an active one in closed systems. In order to improve the investigation one may vary the signals u (or v) which act on the controlled object B. The object is, as it were, "reconnoitered" by signals of an enquiring character whose purpose it is to promote a more rapid and more accurate study of the object's characteristics and of the methods of controlling it.

However, the controlling movements are necessary not only to study or to learn the characteristics of the object or the ways of controlling it, but also to implement the regulation, to direct the object to the required state. Thus, the controlling effects in the general block diagram in Fig. 3 must be twofold: they must, to a certain extent, be investigating as well as directing.

The control whose regulating effects are of this twofold character will in the sequel be called dual control; the papers in the present series will be devoted to the theory of dual control.

Dual control is particularly useful and even indispensible in cases where the operator and the interference z in the object B are complex, and the object is thus distinguished either by its complexity or by the variability of its characteristics. Some typical examples of systems with dual control are to be found in automatic search systems, in particular, in automatic optimization systems (see, for example, [14 and 15]). In these systems, the investigating or "trial" part can usually be separated easily from thecontrolling or "operating" part of the signal, either by the difference in their frequency ranges or because they interweave in time. Such a separation, however, need not always take place; an effect can be twofold in character by virtue of being partly diagnostic and partly regulating.

Thus, in dual-control systems, there is a conflict between the two sides of the controlling process, the investigational and the directional. An efficient control can only be effected by a well-timed action on the object. A delayed action weakens the control process. But the control can only be effective when the properties of the object are sufficiently well known; one needs, however, more time to become familiar with them. A too "hasty" controlling member will carry out the operational movement without making proper use of the results of trial investigations performed on the object. A too "cautious" system will bide its time unnecessarily long and process the received information without directing the object to its required state at the right time. In each case, the control process may not prove the best one and may not even prove to be up to the mark. Our problem is to find out, one way or another, which combination of these two sides of the regulation would prove to be most suitable. The operations must be so selected as to maximize a criterion of the control's quality.

As shown above, the incomplete information about the object will be expressed by the presence of the probability distributions of potentially possible characteristics of the object. The regulating member compares, as it were, the various hypotheses on the object, with probability of its occurrence being attached to each hypothesis. These probabilities vary with time. There may be a control method such that the most probable hypothesis will always be selected and, therefore, assuming that it is valid, the optimum control method will be attained. Such a control system is not generally optimum in the absolute meaning of the word as the complete information on the object has not been utilized. The probability distribution of the different hypotheses extracted from the experiments is distorted as the probability 1 was ascribed to one of them and the probability 0 to others. A better control method will be one whereby the probabilities of all the hypotheses would be taken into account.

The probability distribution of hypotheses will vary with time, the higher probabilities concentrating more and more in the region of those hypotheses which approach the true characteristics of the object. The pace of concentration and, therefore, the success of the subsequent regulating movements, depends on the character of the preceding regulating movements, on how well they have "sounded" the object. Thus, two factors should be taken into account by the controlling member which decides the specific amount of regulating movement at any given moment of time:

(a) The loss occurring in the value of the quality criterion due to the fact that the outcome of the operation at a given moment, and at subsequent moments of time, will cause a deviation of the object either from the required state or from the best attainable one. The average value of this loss shall be called the action risk.

(b) The loss occurring in the value of the quality criterion due to the fact that the magnitude of the controlling action has not proved the best to obtain information on the characteristics of the object; in view of this, the subsequent actions will not be the best posssible ones either. The average value of this loss shall be called the investigation risk.

It will be shown that for a certain class of systems, the total risk will be equal to the sum of the action and investigation risks.

All systems of automatic search (see [14])are characterized by trial actions. Dual control, therefore, is applicable to all systems of automatic search and, in particular, to automatic optimization systems. It can also be applied to other types of closed-loop systems which do not belong to the automatic search class at all. To illustrate the difference between the two types of dual-control systems, a few examples will be given.

Figure 4,a shows a system which operates as follows: the main regulating member A implements the control of object B, either in an open- or in a closed-loop network (the closed one is indicated by a dashed line). The

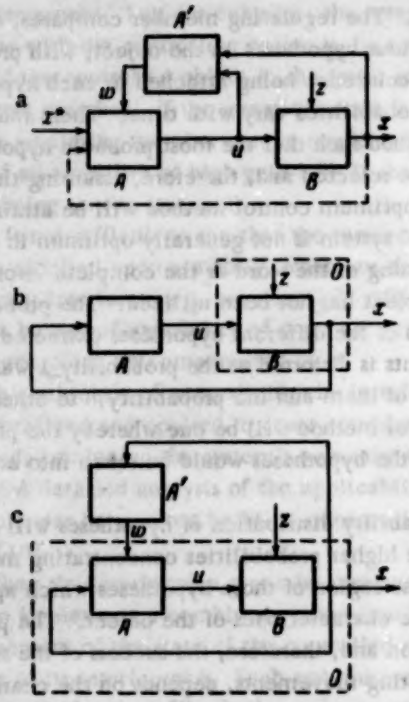

Fig. 4.

regulating movements u are of the investigational and action type simultaneously. The quantities u and x, from the input and the output of object B, respectively, enter an additional controlling member A'. The latter receives the characteristics of object B from the results of the investigations; subsequently, in accordance with an algorithm given in advance and fed into the device from outside, the parameters of controlling member A are so computed that its controlling action is optimal. Having the results of the computation, the additional regulating action w establishes the computed optimum parameters in the main controlling member A. This process may repeat itself periodically.

Such systems contain investigational movements; however, the automatic search is absent. The parameters of member A are established, not via automatic search, but from an algorithm given in advance, from a function of the determined characteristics of object B. There is no investigational component in the operation w. The channel w is not usually found in a closed network, as the change in the coefficients of A has no effect on the coefficients of B.

A block diagram of an automatic optimization system is presented in Fig. 4,b. Here the action u is dual in character, investigating object B as well as directing it to its optimum mode of action. The latter corresponds to an extremum of quantity Q dependent on x. The optimum mode is found by means of automatic search. The latter is conducted in such a way that the information received from the investigating action u and from the output x of the system is analyzed in controlling member A. This permits determination of the regulating part of the same action u on the same object B whose input and output were investigated, so that the same quantity x which was being investigated can be changed in the right direction.

The combination of the investigating and directing operations has, so far, not constituted the whole search, but only one of its distinct features. There is no search in the system depicted in Fig. 4,a; it takes place, however, in the system of automatic optimization shown in Fig. 4,b. But both systems are of the dual-control type.

In Fig. 4,b the controlled object is inside a dashed rectangle denoted by O. In this case, it does not differ from object B. The dual control, however, can also be applied to control the entire automatic system, considered as a complex object. For example, in Fig. 4,c the complex object O inside the dashed rectangle comprises the controlling member A and the object B of control. The auxiliary controlling member A' investigates the process x and, with the aid of the controlling process w, can vary the algorithm of regulation implemented by member A. The w processes are twofold in character. The investigation of changes in the algorithm of member A and their effect on the process x results in regulating processes w, bringing the algorithm of member A to such a form that the process x will either prove admissible, favorable, or optimum, depending for what purpose the system will be used. Here an automatic search takes place in the closed network of processes $w \rightarrow x \rightarrow u$.

Statement of the Synthesis Problem of an Optimum System of Dual Control

The problem of designing an optimum, in the specified meaning of the word, controlling member A, as shown in Fig. 3, is formulated below. It is advisable when formulating the problem to make use of certain concepts of the theory of games and of A. Wald's [4] theory of statistical decisions (see D. Blackwell and M. A. Girshick [16], and also Chow [17]). In solving the variational problem as stated later in the present series of papers, use is made of the concepts of R. Bellman's dynamic programming (see, for example, [18]). In the subsequent parts of this series, the mathematical exposition may appear somewhat cumbersome but is actually quite simple. The main contents of the papers deals with further development of the concepts of automatic control briefly described above.

Consider the scheme presented in Fig. 3. The following limitations of the statement of the problem are introduced.

1) A discrete-continuous system is investigated in which the time but not the level is quantized. All magnitudes occurring in the system are considered at discrete moments of time $t = 0, 1, 2, \ldots, n$ only. Any

magnitude at the sth moment of time will carry the index s. Thus, the considered quantities are x^*_s, x_s, y_s, v_s, g_s, etc.

Such limitation enables one to simplify the computation. Moreover, in many cases this actually occurs. The transition to the continuous time can in some cases be accomplished in an intuitive manner by making the time interval between the discrete values approach zero (see Part III). One meets with considerable difficulties in more fully examining the passage to the limit.

2) The time interval, or the number of cycles n within which the process is being investigated, is assumed to be a fixed constant. In certain cases no major difficulties arise when proceeding to the limit with $n \to \infty$. A wider generalization relating to a variable number n of cycles not known beforehand would be of interest, but will not be tackled in the present paper.

3) A Bayesian problem, in which a priori densities of random variables are given, is considered. Other formulations, for example, minimax, are also of considerable interest, but far more difficult to solve. This problem could also be formulated in relation to the concept of the so-called "inductive probability" (see, for example, the paper of L. S. Schwartz, B. Harris, and A. Hauptschein [19]).

We assume that h^*_s, h_s, g_s, are sequences of independent random variables with identical distribution densities $P(h^*_s)$, $P(h_s)$, $P(g_s)$. Further, let $z_s = z(s, \mu)$, and $x^*_s = x(s, \lambda)$ where μ and λ are random parameter vectors with coordinates μ_i and λ_i, respectively:

$$\mu = (\mu_1, \ldots, \mu_m), \qquad \lambda = (\lambda_1, \ldots, \lambda_q). \tag{4}$$

The a priori probability densities $P(\mu)$ and $P(\lambda^*)$ are given.

4) The object B is assumed to be memoryless; in other words, the values x_s of its output depend only on the values of the input quantities z_s and v_s at the same moment of time:

$$x_s = F_0(z_s, v_s). \tag{5}$$

The functions F_0 and z_s are assumed to be finite and single-valued, continuous and differentiable.

A generalization relating to objects with memory and with x_s depending on x_r, z_r, v_r $(r < s)$ will be given in Part IV. It should be pointed out that memoryless objects are of great practical value. Namely, if the input data (initial conditions or values of parameters) are given for a certain model, and one is able to carry out experiments using this model and also to register the results, then such an object becomes equivalent to a memoryless one.

5) A simple criterion W of quality is introduced.

Let the partial loss function corresponding to the sth time moment be of the form

$$W_s = W(s, x_s, x^*_s). \tag{6}$$

Moreover, let the total loss function W for the total time be equal to the sum of partial loss functions (such a criterion shall be called a simple one):

$$W = \sum_{s=0}^{s=n} W(s, x^*_s, x_s). \tag{7}$$

The smaller the mathematical expectation of W, the better is the system. It shall be called optimum when its average risk R (i.e., the mathematical expectation M of the quantity W) is minimal. The amount of risk is given by the formula

$$R = M\{W\} = \tag{8}$$

$$= M\left\{\sum_{s=0}^{s=n} W(s, x^*_s, x_s)\right\} = \sum_{s=0}^{s=n} M\{W_s\} = \sum_{s=0}^{s=n} R_s.$$

Each $R_s = M\{W_s\}$ will be called a partial risk due to the sth cycle.

There may be many types of simple criteria, for example,

$$W_s = \alpha(s)[x_s - x^*_s]^2. \tag{9}$$

Criteria of practical importance need not always be simple, and generalizations relating to other criteria would therefore be of interest.

The formulation of the optimum strategy problem in terms of risks is not the only one in existence. There exist a number of studies in which closed systems are investigated from the point of view of the information theory (see, for example, R. L. Dobrushin's paper [20]). As the primary aim of a control system does not lie in transmitting information but in designing required processes, the formulation of the problem in the language of statistical decisions fits in better with the intrinsic nature of the problem.

6) All the quantities occurring in the sth cycle will be regarded as scalar. Our object, therefore, has only a single input v and a single output x. The exposition becomes more involved with generalizations relating to objects with several inputs and outputs (see Part IV).

7) We assume that the manner by which the signal and the noise are combined in H*, H, or G blocks is

known and invariable, and that the blocks are memoryless. Thus,

$$v = v(u, g),$$
$$y^* = y^*(h^*, x^*), \qquad (10)$$
$$y = y(h, x).$$

Therefore, the conditional probabilities $P(y^*|x^*)$ and $P(y|x)$ and $P(v|u)$ make sense.

8) We assume that the controlling member A generally possesses a memory and that, moreover, for the sake of generality, the algorithm of its action is a random one, i.e., the part A exhibits random strategy.

We introduce the vectors $(0 \leq s \leq n)$:

$$\mathbf{u}_s = (u_0, u_1, \ldots, u_s),$$
$$\mathbf{y}_s^* = (y_0^*, y_1^*, \ldots, y_s^*), \qquad (11)$$
$$\mathbf{y}_s = (y_0, y_1, \ldots, y_s).$$

The controlling member can now be characterized by the probability densities

$$P_s(u_s) = \Gamma_s(u_s, \mathbf{u}_{s-1}, \mathbf{y}_s^*, \mathbf{y}_{s-1}) \quad (0 \leqslant s \leqslant n). \qquad (12)$$

The problem consists in finding a sequence of functions F_s such that the average risk R (see [8]) becomes minimal.

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PARAMETRIC PHENOMENA IN THE SIMPLEST CONTINUOUS EXTREMAL CONTROL SYSTEMS

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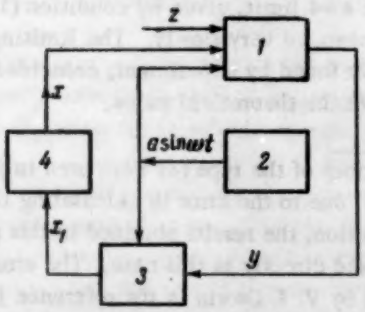
This article presents an analysis of a continuous control system where the extremum of the characteristic of an inertialess system to be controlled is searched for and maintained in the presence of intensive random noise in the frequency band which includes the frequency of the periodic trial input. The stability conditions for this system are determined.

The spectrum of uncontrolled external signals acting on an extremal control system usually contains a low-frequency band, which is determined by the drift of elements in the system, and a low-frequency band determined by noise. Between these bands lies a frequency band which is almost free from noise. However, this third band may be insufficiently broad to serve for assigning the frequency of trial inputs [1, 2]. Therefore, the case wherein the trial input frequency lies within the noise band is of especial interest. However, if we assume that the high-frequency noise appears only at the output of the system to be controlled, no basic difficulties are encountered in analyzing the equations of the closed-loop system [2]. For a controlled system whose characteristic is given by a quadratic parabola, the control system is described by linear equations with constant coefficients. For an arbitrary characteristic, the problem of determining the main deviation of the system operating conditions from the extremum can also be solved by using the statistical linearization method. However, if high-frequency noise appears at the system input, the coefficients in the equation for the closed-loop system become random functions of time. The investigation of such equations is a complicated task.



1) System to be controlled; 2) generator of trial oscillations; 3) demodulator; 4) final mechanism.

An inertialess system of the simplest form with high-frequency noise at the input of the system to be controlled is considered below. The block diagram of this system is given in the figure.

If the characteristic of the system to be controlled is given by the quadratic parabola $y=-x^2$, the system operation will be described by the following equations:

1) The equation of the system to be controlled,

$$y = -(x + a\sin\omega t + z)^2,$$

where a sin ωt is the trial input, and z is the noise;

2) the demodulator equation,

$$x_1 = \frac{b\omega}{2\pi}\int_t^{\frac{2\pi}{\omega}+t} y\sin\omega t\,dt; \qquad (1)$$

3) the final mechanism equation,

$$x - x_0 = \int_0^t x_1\,dt.$$

We shall limit our consideration to the case wherein the spectrum z(t) of the normal random noise has a narrow band and the noise can, consequently, be represented in the form

$$z(t) = A(t)\sin\omega t + C(t)\cos\omega t. \qquad (2)$$

Here, A(t) and C(t) are slowly changing (in comparison with the trial input) random processes. By taking into account condition (2), system (1) is reduced [2] to the equation

$$\frac{dx}{dt} + b[a + A(t)]x = 0 \qquad (3)$$

or

$$\frac{dx}{dt} + \xi(t)x = 0^*, \qquad (4)$$

while

$$m_\xi = b(a + m_A), \quad B_\xi(\tau) = b^2 B_A(\tau), \qquad (5)$$

where m_A, B_A, m_ξ, and B_ξ are the mathematical expectations and the autocorrelation functions of the A(t) and ξ(t) processes, respectively.

Thus, the presence of high-frequency noise at the input of the system to be controlled causes the appearance of a parameter which changes in random fashion in time in the equation which determines the variable x of the control mechanism. According to this method [3], after correcting the error contained therein, we shall obtain the following equations for the mathematical expectation and dispersion:

$$m_x(t) = x_0 \exp\left\{-\left|m_\xi - \int_0^{t-t_0} B_\xi(\tau)\,d\tau\right|(t-t_0)\right\} \times$$

$$\times \exp\left[-\int_0^{t-t_0} \tau B_\xi(\tau)\,d\tau\right], \qquad (6)$$

$$\sigma_x^2(t) = x_0^2 \exp\left\{-2\left|m_\xi - 2\int_0^{t-t_0} B_\xi(\tau)\,d\tau\right|(t-t_0)\right\} \times$$

$$\times \exp\left|-4\int_0^{t-t_0} \tau B_\xi(\tau)\,d\tau\right| -$$

$$-x_0^2 \exp\left\{-2\left[m_\xi - \int_0^{t-t_0} B_\xi(\tau)\,d\tau\right](t-t_0)\right\} \times$$

$$\times \exp\left[-2\int_0^{t-t_0} \tau B_\xi(\tau)\,d\tau\right]. \qquad (7)$$

Here, $x = x_0$ for $t = t_0$.

The values of m_x and σ_x for $t \to \infty$ obviously depend on the sign in front of the quantities:

$$\lambda = m_\xi - \int_0^\infty B_\xi(\tau)\,d\tau, \quad \mu = m_\xi - 2\int_0^\infty B_\xi(\tau)\,d\tau. \qquad (8)$$

Since $T = \int_0^\infty B_\xi(\tau)\,d\tau > 0$ and $\mu = \lambda - T$, the following variants can be realized:

I. $\lambda > 0$, $\mu > 0$;
II. $\lambda > 0$, $\mu = 0$;
III. $\lambda > 0$, $\mu < 0$;
IV. $\lambda = 0$, $\mu < 0$;
V. $\lambda < 0$, $\mu < 0$. (9)

In this connection, the following cases can occur under steady-state conditions:

I. $m_x(\infty) = 0$, $\sigma_x^2(\infty) = 0$ for any initial conditions;

II. $m_x(\infty) = 0$ for any initial condition, and

$$\sigma_x^2(\infty) = x_0^2 \exp\left[-4\int_0^\infty \tau B_\xi(\tau)\,d\tau\right];$$

III. $m_x(\infty) = 0$ for any initial condition; for $x_0 \neq 0$, $\sigma_x^2(t)$ increases boundlessly with time or, in other words, the system is unstable with respect to dispersion;

IV. $m_x(\infty) = x_0 \exp[-\int_0^\infty \tau B_\xi(\tau)\,d\tau]$, and the system is unstable with respect to dispersion;

V. The system is unstable with respect to the average value as well as with respect to dispersion.

Hence, it is obvious that the dispersion is limited under steady-state conditions if $\mu \geq 0$. For a system described by Eq. (3), this condition is of the following form:

$$b(a + m_A) - 2b^2 \int_0^\infty B_A(\tau)\,d\tau \geqslant 0 \qquad (10)$$

or, for $b > 0$,

$$a \geqslant 2b\tau_A \sigma_A^2 - m_A. \qquad (11)$$

Here, $\sigma_A^2 = B_A(0)$ is the dispersion of the random process A(t), and $\tau_A = [1/B_A(0)]\int_0^\infty B_A(\tau)\,d\tau$ is the time of the correlation process A(t).

These conclusions were verified by experiment.† For b = 1, Eq. (3) was reproduced by means of an electronic simulating device. The random noise was secured by a low-frequency noise generator. In this, the correlation time was $\tau_A = 0.25$ sec, and $\sigma_A^2 = 8$ v.

A decrease in the a parameter, which corresponded to the amplitude of trial oscillations, led to "stability" losses. In this, as was to be expected from relation (7), which was written for Eq. (3), the "stability" disturbance took place aperiodically, i.e., for a small deviation from the a = 4 limit, given by condition (11), the dispersion σ_x^2 changed very slowly. The limiting value itself, which was found by experiment, coincided almost exactly with the theoretical value.

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* Equations of the type (4) were used in paper [3]. However, due to the error in calculating the characteristic function, the results obtained in this paper could not be used directly in this case. The error in [3] was detected by V. I. Osorin in the reference journal Mekhanika 10, 11232 (1959).

† E. P. Gil'bo performed the experiment.

Thus, the stability loss in scanning and in maintaining the extremum, caused by the presence of high-frequency noise at the input of the system to be controlled, is entirely possible. This limits the possible amplitudes of trial oscillations and, consequently, the maximum allowable scanning loss.

Regardless of the simplicity of the problem as formulated at the outset, its investigation made it possible to obtain a definite physical result which can be applied to all continuous extremal systems operating in accordance with the gradient method. It consists in the fact that the consideration of random noises leads to the necessity of a more complicated approach to the stability problem in such systems than was used, for instance, in [4], where the entire problem was reduced to the investigation of linear systems with constant coefficients.

If the noise level at the input of the system to be controlled is high, the following of the extremum may be impossible due to the noise parametric action.

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EFFECT OF FLUCTUATIONS ON EXTREMAL RELAY SYSTEMS UNDER SELF-OSCILLATION CONDITIONS

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This article presents an analysis of extremal relay control systems under self-oscillation conditions with fluctuations at the regulator input. A method of statistical linearization of such systems, based on the reduction of external random disturbances to parametric action on the regulator, is proposed. This method makes it possible to obtain an expression for the equivalent gain in the form of a random step function of time according to the describing-function method. The average value of this function is taken as the statistically equivalent transfer coefficient of the nonlinear component.

This method can be applied to ordinary nonlinear self-oscillation systems.

In extremal relay systems (ERCS), the self-oscillation mode is the operating mode. Under operating conditions, an actual system is usually subject to the action of external random disturbances. The present work analyzes the behavior of ERCS under self-oscillation conditions, when $\xi(t)$ fluctuations are applied to the regulator, input. The investigation is based on the statistical-linearization method in conjunction with the describing-function method applied in the form developed for extremal systems in [1].

The use of the statistical-linearization method [2] in the investigation of systems allowing stable self-oscillation states was first proposed in [3]. In [2] and [3], by operating with the first moments of random functions, the linear (equivalent in the probability sense) relation between the input and output moments of the function at the input and output of the nonlinear component is determined on the basis of a certain criterion, while, in [3], this is achieved by using the describing-function method. This method cannot be applied to extremal relay systems. This is due to the fact that, for random disturbances at the extremal regulator input, the parameter of this regulator (the relay threshold) becomes a function of the instantaneous value of the external random disturbance. Therefore, for nonlinearities of this type, the consideration cannot be limited to the random function moments at the regulator input.

Another statistical-linearization method is proposed in the present paper. It is based on a nonlinear reduction of the random function to a form suitable for introducing the regulator fluctuating parameter in the expression for the equivalent gain of the nonlinear element, which is obtained according to the describing-function method. Thus, if, in [3], the equivalent coefficient is obtained as a function of the first moments of the external random disturbance, here it represents a random function of time. Its average value is found with respect to the distribution density of the determined fluctuating parameter function.

The results obtained in investigating ERCS in accordance with the above-mentioned idea for the system linearization are given below.*

1. Formulation of the Problem and the Method of Solution

Let $\xi(t)$ represent a steady-state normal random process with zero average value and intensity σ_ξ, which is smaller than the amplitude Y of self-oscillations in the system, and let the basic components of the fluctuation spectrum $\xi(t)$ be comparable to the frequency ω of self-oscillations. Such a statement of the problem is of practical value since the high-frequency fluctuation components can always be suppressed by means of a weak filter in the control circuit without affecting the dynamic properties of the system.

As is known, ERCS have two basically nonlinear inertialess components. In approximating the static characteristic $y = f_1(x)$ of the system to be controlled by a quadratic parabola, due to the smallness of σ_ξ the system to be controlled can be represented in the form of a linear element with the equivalent transfer locus

* In connection with another problem, a statistical-linearization method, conceptually close to the method mentioned above and based on different physical assumptions, was proposed by A. A. Pervozvanskii in [8].

$K(j\omega)$ in investigating self-oscillations [1]. The second nonlinear component —the controlling device with $z=f_2(y)$— has the ratchet-relay characteristic [4] shown in Fig. 1.

This characteristic can be described by the expression

$$z = \begin{cases} \pm k_p \text{ for } y \lessgtr 0, & \dot{y} > 0; \\ \pm k_p \text{ for } y + \varkappa_1 > 0, \\ \mp k_p \text{ for } y + \varkappa_1 < 0, & \dot{y} < 0. \end{cases} \quad (1)$$

Here, k_p is the relay gain (the choice of the sign sequence depends on the initial conditions) and $\varkappa_1 = \varkappa - y^*$ is the regulator fluctuating parameter, which is expressed in terms of $\varkappa$ (quantization step of the output quantity with respect to level) and y^*, which depends on the system type.

Let the input quantity of the control device be expressed by the sum

$$y = Y \sin \omega t + \xi(t). \quad (2)$$

Then

$$y^* = Y + \xi(t)_{t=\frac{\pi}{2\omega}} \quad (3)$$

for ERCS with a floating quantization grid, and

$$y^* = [Y \sin \omega t + \xi(t)]_{\max} \quad (3a)$$

for ERCS with a fixed grid of the output quantity quantization with respect to level in the system to be controlled [5]. According to (1), we shall write the condition for the pertinent switching moment in the system:

$$y(t_n) + \varkappa_1(t_n) = 0. \quad (4)$$

By substituting (2) in (4), we can reduce the random component $\xi(t)$ of the control device input to the fluctuating parameter $\varkappa_1$ and thus represent the effect of an external random disturbance on the system only in the form of a parametric action on the regulator.

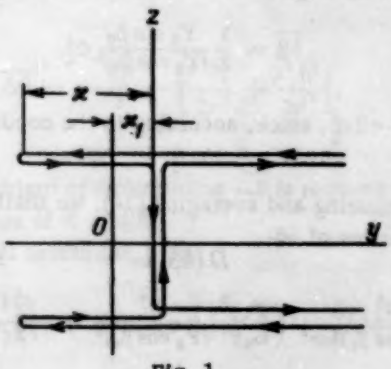

Fig. 1.

Since the actuation phase β of the relay enters the expression for the relay regulator equivalent gain, the fluctuating parameter of this regulator can be conveniently transformed into the fluctuating actuation phase $\delta\beta$ of the relay for a regulator sinusoidal input. As is shown below, the final results can thus be geometrically interpreted in a clear manner. Thus, the second stage in solving the stated problem consists in the nonlinear transformation of the relay parameter random function into the random function $\delta\beta$, with the ultimate aim of determining the distribution density of the new random parameter $\delta\beta$.

Assuming that $\delta\beta = \text{const} \neq 0$ during the period T of the regular input first harmonic, the equivalent gain $S(Y, \delta\beta)$ for the given period can be obtained. It should be noted that this operation is valid if the self-oscillations in the system are settled within a time interval which is less than the period T. As is shown below, in ERCS this is practically always the case.

In the final stage, the average value $\overline{S(Y, \delta\beta)}$ is determined, after which, by plotting the graphic solution in the usual form given by the describing-function method, we can find the average values of the self-oscillation regime characteristics in ERCS when the above $\xi(t)$ fluctuations act at the regulator input.

2. Distribution of the ERCS Relay Actuation Phase under Self-Oscillation Conditions in the Presence of Fluctuations

We shall first find the static characteristics of the phase distribution for an ERCS with a floating grid of output quantization with respect to level.

The phase distribution will be expressed in terms of angles ωt, where ω is the periodic regime frequency at the output of the extremal system to be controlled.

In the absence of fluctuations, the relay actuation phase β_0 will be determined by the moment when the $Y_0 \sin \omega_0 t$ signal intersects a certain level $\varkappa$, which is measured from the Y_0 value, i.e.,

$$Y_0 \sin \beta_0 = \varkappa - Y_0, \quad (5)$$

where [5] $Y < \varkappa \leq 2Y$.

For a random external input, considering that the $\xi(t)$ fluctuations enter the control signal linearly, we shall seek the phase β^* of the sinusoidal component at the moment when the $\xi(t_2) + Y_0 \sin \beta^*$ signal intersects the level measured from $Y_0 + \xi(t_1)$:

$$\xi(t_2)_{t_2=\frac{\beta^*}{\omega}} + Y_0 \sin \beta^* = \varkappa - [Y_0 + \xi(t_1)]. \quad (6)$$

By eliminating (5) from (6), we obtain

$$Y_0(\sin \beta^* - \sin \beta_0) + \xi(t_1) + \xi(t_2) = 0. \quad (7)$$

In order to determine the statistical characteristics of the random function $\delta\beta$ according to Eq. (7), we shall use the solution which has been proposed in [6].

Let us define more accurately the above restrictions which have been imposed on $\xi(t)$. We shall consider that $\xi(t)$ is a centered function with a Gaussian distribution and the autocorrelation function $\sigma_\xi^2 R(\tau)$, where

$$\sigma_\xi < Y \tag{8}$$

and the correlation time

$$\overline{\frac{\beta^* - \beta_0}{\omega}} < \tau_k < \frac{\pi}{2\omega} \tag{9}$$

throughout the entire working range of the frequencies ω.

Condition (9) permits us to consider that the first intersection, the one in which we are interested, is also the only one; this condition also determines the independence of the $\xi(t_1)$ and $\xi(t_2)$ values of the random function $\xi(t)$.

Under the assumption that the fluctuations are small, we shall introduce a small parameter μ, by denoting in correspondence with (9):

$$\xi(t_1) + \xi(t_2) = \zeta(t_2) = \mu\zeta^*\left(\frac{\beta^*}{\omega_0}\right), \tag{10}$$

and we write the initial equation (7) in the form

$$Y_0(\sin\beta^* - \sin\beta_0) + \mu\zeta^*\left(\frac{\beta^*}{\omega_0}\right) = 0. \tag{11}$$

According to [6], we shall seek the solution of this equation in the form of a power series of μ

$$\beta^* = \beta_0 + \mu\beta_1 + \mu^2\beta_2 + \cdots \tag{12}$$

By substituting β^* in (11), we obtain

$$Y_0\{\sin(\beta_0 + \mu\beta_1 + \mu^2\beta_2 + \ldots) -$$

$$- \sin\beta_0\} + \mu\zeta^*\left[\frac{1}{\omega_0}(\beta_0 + \mu\beta_1 + \mu^2\beta_2 + \ldots)\right] = 0.$$

We expand the functions of the β^* variable in a Taylor series with respect to β_0:

$$Y_0\left\{(\mu\beta_1 + \mu^2\beta_2 + \ldots)\cos\beta_0 -\right.$$

$$\left. - \frac{1}{2}(\mu\beta_1 + \mu^2\beta_2 + \ldots)^2 \sin\beta_0 + \ldots\right\} + \mu\zeta^*\left(\frac{\beta_0}{\omega_0}\right) +$$

$$+ \frac{\mu}{\omega_0}\zeta^{*\prime}\left(\frac{\beta_0}{\omega_0}\right)\left[\frac{1}{\omega_0}(\mu\beta_1 + \mu^2\beta_2 + \ldots)\right] + \ldots = 0.$$

By equating the coefficients in front of the equal powers of the μ parameter, we arrive at the following system of equations:

$$\beta_1 Y_0 \cos\beta_0 + \zeta^*\left(\frac{\beta_0}{\omega_0}\right) = 0,$$

$$\beta_2 Y_0 \cos\beta_0 - \beta_1^2 \frac{1}{2} Y_0 \sin\beta_0 + \beta_1 \frac{1}{\omega_0^2}\zeta^{*\prime}\left(\frac{\beta_0}{\omega_0}\right) = 0,$$

. . . . . ,

whence we find the successive approximations for $0 < \beta_0 \leq (\pi/2)$ (the region wherein self-oscillations exist [5]):

$$\beta_1 = -\frac{\zeta^*\left(\frac{\beta_0}{\omega_0}\right)}{Y_0\cos\beta_0},$$

$$\beta_2 = \frac{\zeta^*\left(\frac{\beta_0}{\omega_0}\right)\zeta^{*\prime}\left(\frac{\beta_0}{\omega_0}\right)}{\omega_0^2 Y_0^2 \cos^2\beta_0} + \frac{1}{2}\frac{\zeta^{*2}\left(\frac{\beta_0}{\omega_0}\right) Y_0 \sin\beta_0}{(Y_0\cos\beta_0)^3}.$$

Consequently, the solution of Eq. (11) with an accuracy to μ^2 can be represented by the expression

$$\beta^* - \beta_0 = \delta\beta = -\mu\frac{\zeta^*\left(\frac{\beta_0}{\omega_0}\right)}{Y_0\cos\beta_0} +$$

$$+ \mu^2\left\{\frac{\zeta^*\left(\frac{\beta_0}{\omega_0}\right)\zeta^{*\prime}\left(\frac{\beta_0}{\omega_0}\right)}{(\omega_0 Y_0\cos\beta_0)^2} + \frac{1}{2}\frac{\zeta^{*2} Y_0\sin\beta_0}{(Y_0\cos\beta_0)^3}\right\}. \tag{13}$$

According to (10), due to the independence of $\xi(t_1)$ and $\xi(t_2)$ and the steady-state property of $\zeta(t)$, we finally write (13) in the form

$$\delta\beta = -\frac{\zeta}{Y_0\cos\beta_0} + \frac{\zeta\zeta'}{(\omega_0 Y\cos\beta_0)^2} + \frac{1}{2}\frac{\zeta^2 Y_0\sin\beta_0}{(Y_0\cos\beta_0)^3}. \tag{14}$$

The $\delta\beta(t)$ function has a physical meaning only for instants of time t_n which coincide with the times of observing the β^* phase of the first harmonic of the regular input, and it is equal to $\delta\beta(t_n)$ for any instant of time within the $t_n \leq t < t_{n+1}$ interval.

Let us find some values of the statistical characteristic $\delta\beta$. By averaging (14) with respect to t_n, we obtain the following average value:

$$\overline{\delta\beta} = \frac{1}{2}\frac{Y_0\sin\beta_0}{(Y_0\cos\beta_0)^3}\sigma_\zeta^2, \tag{15}$$

where $\sigma_\zeta^2 = 2\sigma_\xi^2$, since, according to the condition $\overline{\xi^2} = 0$, $\overline{\xi\xi'} = 0$.

By squaring and averaging (14), we shall find [6] the dispersion of $\delta\beta$:

$$D(\delta\beta) =$$

$$= \frac{\sigma_\zeta^2}{(Y_0\cos\beta_0)^2} + \left(\frac{\omega_\zeta}{\omega_0}\right)^2\frac{\sigma_\zeta^4}{(Y_0\cos\beta_0)^4} + \frac{\sigma_\zeta^4(Y_0\sin\beta_0)^2}{2(Y_0\cos\beta_0)^6},$$

where $\omega_\zeta^2 = -(d^2R/d\tau^2)_{\tau=0}$ denotes the root-mean-square value of the frequency of the energy spectrum of fluctuations ζ. If condition (8) is satisfied, we can use only the first term in the above equation:

$$D(\delta\beta) = \frac{\sigma_\zeta^2}{Y_0^2}\,\frac{1}{\cos^2\beta_0}. \qquad (16)$$

The characteristic function $\delta\beta$ serves for determining† [6] the distribution density $w(\delta\beta)$:

$$w(\delta\beta) = \left[1 - \delta\beta\frac{\sin\beta_0}{\cos\beta_0} + (\delta\beta)^3\frac{Y_0^2}{2\sigma_\xi^2}\sin\beta_0\cos\beta_0\right]\frac{Y_0}{\sigma_\zeta}\,\frac{\cos\beta_0}{\sqrt{2\pi}}\,e^{-\frac{(\delta\beta)^2}{2\sigma_\zeta^2}Y_0^2\cos^2\beta_0},$$

or, by taking into account the smallness of σ_ζ,

$$w(\delta\beta) = (1 - \delta\beta\,\mathrm{tg}\,\beta_0)\,\frac{Y_0}{\sigma_\zeta}\,\frac{\cos\beta_0}{\sqrt{2\pi}}\,e^{-\frac{1}{2}\left(\frac{\delta\beta}{\sigma_\zeta}\right)^2 Y_0^2\cos^2\beta_0}. \qquad (17)$$

For ERCS having a fixed quantization grid, the calculation of the same statistical characteristics of the relay actuation phase distribution $\Delta\beta$ is considerably more complicated. Therefore, we shall calculate only the average value $\overline{\Delta\beta}$ for this case.

By analogy with (7) and on the basis of (3a), we obtain the initial equation in the following form:

$$Y_0(\sin\beta^* - \sin\beta_0) + [Y_0\sin\omega_0 t_1 + \xi(t_1)]_{\max} + \xi(t_2) - 0. \qquad (18)$$

After denoting $s = Y_0\sin\beta_0$ and $\eta = [Y_0\sin\omega_0 t + \xi(t)]_{\max}$, we shall use solution (14) for $\zeta = \xi + \eta$:

$$\Delta\beta = \frac{\xi+\eta}{s'} + \frac{(\xi+\eta)(\xi'+\eta')}{\omega_0^2 s'^2} - \frac{s''(\xi+\eta)^2}{2s'^3}.$$

Hence, for $\overline{\xi} = 0$ and $\overline{\xi'} = 0$, and mutually independent values of ξ, ξ', η, and η', we obtain:

$$\overline{\Delta\beta} = -\frac{\overline{\eta}}{s'} - \left[\frac{s''\sigma_\xi^2}{2s'^3} + \frac{s''\overline{\eta^2}}{2s'^3}\right], \qquad (19)$$

i.e., the problem of determining $\overline{\Delta\beta}$ is reduced to the determination of $\overline{\eta}$ and $\overline{\eta^2}$.

We shall determine η as

$$\eta = s(\gamma + \delta\gamma) - s(\gamma) + \xi\left(\frac{\gamma+\delta\gamma}{\omega_0}\right), \qquad (20)$$

where γ corresponds to the extremum of ξ, and $\delta\gamma$ is determined from the condition

$$s'(\gamma+\delta\gamma) + \xi'\left(\frac{\gamma+\delta\gamma}{\omega_0}\right) = 0. \qquad (21)$$

Then

$$\overline{\eta} = \overline{s(\gamma+\delta\gamma)} - s(\gamma) = \overline{s(\delta\gamma)}, \qquad (22)$$

$$\overline{\eta^2} = [s(\delta\gamma) + \xi(\gamma+\delta\gamma)]^2 = \overline{s^2(\delta\gamma)} + \sigma_\xi^2, \qquad (23)$$

since $s(\delta\gamma)$ is a function of ξ', i.e., it is independent of ξ.

Thus, having determined, as before, the distribution density δ_γ from solution (21), we can calculate $\overline{\eta}$ and $\overline{\eta^2}$ and, consequently, also $\overline{\Delta\beta}$. In the final form,

$$\overline{\Delta\beta} = \frac{\sigma_\xi}{Y_0^2}\,\frac{\sin\beta_0}{\cos^2\beta_0} - \left(\frac{\omega_\xi}{\omega_0}\right)^2\frac{\sigma_\xi^2}{2Y_0^2}\,\frac{1}{\cos\beta_0}$$

or

$$\overline{\Delta\beta} = \overline{\delta\beta} - \left(\frac{\omega_\xi}{\omega_0}\right)^2\frac{\sigma_\xi^2}{2Y_0^2}\,\frac{1}{\cos\beta_0}. \qquad (24)$$

The determination of higher statistical characteristics $\Delta\beta$ involves considerable difficulties in calculation and requires the knowledge of higher $\xi(t)$ moments.

3. Determination of the ERCS Regulator Equivalent Gain in the Case of Parametric Random Action

We shall determine the time t_{ss} necessary for the self-oscillations to reach the steady-state in an ERCS with respect to the self-oscillation period T in the system. From the condition for the existence of self-oscillations in ERCS [5], it follows that $T > T_{cr} = (2\pi/\omega_{cr})$, where ω_{cr} is the maximum value of the self-oscillation frequency, which is determined by the equation

$$2\theta_1\left(\frac{\omega_{cr}}{2}\right) + \theta_2(\omega_{cr}) = -\frac{\pi}{2}. \qquad (25)$$

Here, θ_1 and θ_2 are the phase responses of linear factors in the equivalent diagram of the extremal system to be controlled before and after the nonlinear static characteristic, respectively.

In the majority of cases of practical interest, the inertial portion of the system to be controlled can be represented by a connection in series of first-order factors. In this, the transient process time can be approxi-

† By taking only the first term in the expansion of the characteristic function $\overline{e^{jv\delta\beta}}$ with respect to $\delta\beta$.

mately determined with respect to the interval ω_p, where the real part of the transfer locus of the system inertial portion is positive:

$$\theta_1(\omega_p) + \theta_2(\omega_p) = \frac{\pi}{2}. \quad (26)$$

For any number of first-order factors,

$$t_{ss} < \frac{2\pi}{\omega_p}.$$

From a comparison between (25) and (26), it follows that ω_{cr} is always smaller than ω_p, i.e.,

$$t_{ss} < \frac{2\pi}{\omega_p} < \frac{2\pi}{\omega_{cr}} < T. \quad (27)$$

Thus, it can be said that self-oscillations in ERCS attain the steady-state in a time which is less than one period in all cases of practical interest. This fact makes it possible to determine, by using the describing function method, the equivalent complex gain $S(Y, \delta\beta)$ of the control device as a function of $\delta\beta(t_n)$, assuming that $\delta\beta = \text{const}$ during the averaging period $t_{n+1} - t_n$. In this, $S(Y, \delta\beta)$ will always represent a random step function of time instants t_n. If we know the $\delta\beta$ distribution, we can determine the average value of the $S(Y, \delta\beta)$ function.

For ERCS of the type under discussion, $S(Y, \delta\beta)$ in exponential form for $\delta\beta \neq 0$ is expressed by [1]

$$S(Y, \delta\beta) = \frac{4k_p}{\pi Y_0} e^{j(\beta_0 + \delta\beta)}$$

or, for $\beta_0 = \pi - \alpha_0$ [1] and $\delta\beta = \delta\alpha$, by

$$\frac{1}{S(Y, \delta\alpha)} = N(Y, \delta\alpha) = -\frac{\pi Y_0}{4k_p} e^{j(\alpha_0 - \delta\alpha)}. \quad (28)$$

The average value is given by

$$\overline{N(Y, \delta\alpha)} = \int_{-\infty}^{\infty} N(Y, \delta\alpha)\, w(\delta\alpha)\, d(\delta\alpha). \quad (29)$$

After substituting in (29) the expressions for the integrands from (28) and (17) and after integration (see [7]), we obtain

$$\overline{N(Y, \delta\alpha)} = \frac{\pi Y_0}{4k_p} e^{j\alpha_0}\left[1 + j2\frac{\sigma_\xi^2}{Y_0^2}\frac{\sin\alpha_0}{\cos^3\alpha_0}\right] e^{-\frac{\sigma_\xi^2}{Y_0^2\cos^2\alpha_0}}. \quad (30)$$

We shall expand $\exp(-\sigma_\xi^2/Y_0^2\cos^2\alpha_0)$ in a power series of $(\sigma_\xi^2/Y_0^2\cos^2\alpha_0)$ and, by using only the linear approximation, we shall reduce (30) to the form:

$$\overline{N(Y, \delta\alpha)} = N(Y, 0) + \overline{\delta N(Y, \delta\alpha)}, \quad (31)$$

where $N(Y, 0) = (-\pi Y_0/4k_{cr}) e^{j\alpha_0}$ is the inverse of the equivalent gain in the absence of a random parametric action, and $\overline{\delta N(Y, \delta\alpha)}$ is the average value of the correction brought forth by fluctuations.

By developing the $N(Y, 0)$ and $\overline{\delta N(Y, \delta\alpha)}$ expressions for $\alpha_0 = \arcsin[(\varkappa - Y_0)/Y_0]$ [1], after simple transformations we finally obtain the following expression suitable for graphic construction:

$$N(Y, 0) = -\frac{\pi}{4k_p}\left[\sqrt{2\varkappa Y_0 - \varkappa^2} + j(\varkappa - Y_0)\right], \quad (32)$$

$$\overline{\delta N(Y, \delta\alpha)} = \frac{\pi\sigma_\xi^2}{4k_p}\left[\frac{(\varkappa - Y_0)^2 + Y_0^2}{\sqrt{(2\varkappa Y_0 - \varkappa^2)^3}} - j\frac{(\varkappa - Y_0)}{\sqrt{(2\varkappa Y_0 - \varkappa^2)^3}}\right]. \quad (33)$$

The conditions for the existence of periodic solutions in the system are determined by the equation

$$K(j\omega) = N(Y, 0) + \overline{\delta N(Y, \delta\alpha)}, \quad (34)$$

where $K(j\omega)$ is the equivalent transfer locus of the extremal system to be controlled [1]. The graphic construction in Fig. 2 gives an example of the fulfillment of condition (34), as a result of which the average values of the self-oscillation frequency and amplitude in the system are determined.

As was indicated above, for ERCS with a fixed quantization grid, the derivation of the expression for the distribution density $\Delta\beta$ is difficult, and the final solution of the problem cannot be obtained. We shall limit our analysis here to the following qualitative considerations.

Consider the expression for the average deviation of the relay actuation phase,

$$\overline{\Delta\beta} = \overline{\Delta\alpha} = \overline{\delta\alpha} - \frac{\sigma_\xi^2}{2Y^2}\frac{\omega_\xi^2}{\omega_0^2}\frac{1}{\cos\alpha_0} \quad (35)$$

Let us introduce the $R(\tau) = e^{-(\beta^2/4)\tau^2}\cos\bar{\omega}\tau$ function.

Then,

$$\omega_\xi^2 = -\left(\frac{d^2R}{d\tau^2}\right)_{\tau=0} = \bar{\omega}^2 + \frac{\beta^2}{2}, \quad (36)$$

where $\bar{\omega}$, as is known, denotes the average value of the fluctuation energy spectrum frequency, and β denotes the average spectrum width.

Expression (35), together with (36), indicates that $\overline{\Delta\alpha} < \overline{\delta\alpha}$ always holds and that, for some values of ω_ξ^2,

$\overline{\Delta\alpha}$ can change its sign. This means that for random external disturbances applied to the regulator input of an ERCS having a fixed quantization grid, the self-oscillation frequency for a given σ_ξ is lower than in the preceding system and that, if $\overline{\Delta\alpha} < 0$, it can be even lower than the self-oscillation frequency ω_0 of an undisturbed system.

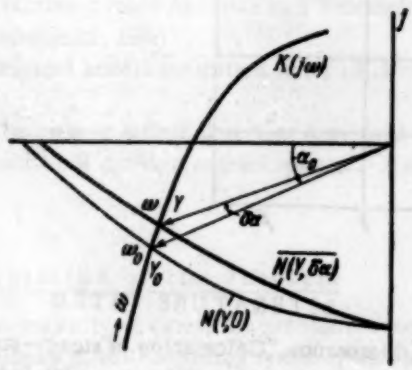

Fig. 2.

4. Certain Generalizations and Comparative Characteristics of the Statistical-Linearization Method under Consideration

The statistical-linearization method which has been considered in application to ERCS can be even more readily applied to ordinary self-oscillation systems having relay characteristics of any type if, naturally, the restrictions imposed on $\xi(t)$ are maintained. Since a random external disturbance $\xi(t)$ applied to the relay component input in a self-oscillation system is always reduced to the random distribution of the relay-actuation phase, the equivalent gain $S(Y, \delta\alpha)$ of a nonlinear element can be found as a step function with a random amplitude distribution according to the usual rules of the describing-function method.

The average value of $\overline{S(Y, \delta\alpha)}$ is determined once and for all with respect to the found expression for distribution (17); this determination is reduced to the calculation of integrals of the type

$$\int_{-\infty}^{\infty} e^{-hx^2-jqx} x^n dx = j^n \frac{d^n\left(e^{-q^2/4h}\right)}{dq^n}, \quad (37)$$

which are calculated according to the method indicated in [7].

For instance, let us find the $\overline{S(Y, \delta\alpha)}$ expression for a relay with the insensitivity zone 2a. As is known, in this case,

$$S(Y, 0) = \frac{4}{\pi Y}\cos\alpha, \quad (38)$$

where $\alpha = \arcsin(a/Y)$, and the relay gain k_p is assumed to be equal to unity for the sake of simplicity.

Then

$$S(Y, \delta\alpha) = \frac{4}{\pi Y}\cos(\alpha + \delta\alpha)$$

and

$$\overline{S(Y, \delta\alpha)} = \int_{-\infty}^{\infty} S(Y, \delta\alpha)\, w(\delta\alpha)\, d(\delta\alpha), \quad (39)$$

where

$$w(\delta\alpha) = (1 - \delta\alpha \operatorname{tg}\alpha)\frac{Y}{\sigma}\frac{\cos\alpha}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\delta\alpha}{\sigma}\right)^2 Y^2\cos^2\alpha}$$

[see (17)].

Let us consider some particular cases.

1. Let $a = 0$ (the relay has no insensitivity zone). For this case, we obtain

$$S(Y, \delta\alpha) = \frac{4}{\pi Y}\cos\delta\alpha, \quad (40)$$

$$w(\delta\alpha) = \frac{1}{\sqrt{2\pi}}\frac{Y}{\sigma} e^{-\frac{1}{2}\left(\frac{\delta\alpha}{\sigma}\right)^2 Y^2}, \quad (41)$$

i.e., we obtain a Gaussian distribution.

By calculating the integral (39) for $a = 0$, we obtain

$$\overline{S(Y, \delta\alpha)} = \frac{4}{\pi Y} e^{-\frac{\sigma^2}{2Y^2}} = \frac{4}{\pi Y} e^{-\frac{1}{4\beta^2}}, \quad (42)$$

where $\beta = (Y/\sqrt{2}\sigma)$.

2. For $a \neq 0$, the $w(\delta\alpha)$ distribution is nonsymmetrical, leading to a complex expression for $\overline{S(Y, \delta\alpha)}$. Omitting the intermediate derivation steps, we shall write the result obtained in calculating integral (39) in the following form:

$$\overline{S(Y, \delta\alpha)} = \frac{4}{\pi Y}\cos\alpha\, e^{-\frac{1}{4\beta^2\cos^2\alpha}} - \left(\operatorname{tg}\alpha + j\right)\frac{1}{2\beta^2}\frac{\operatorname{tg}\alpha}{\cos\alpha} e^{-\frac{1}{4\beta^2\cos^2\alpha}}. \quad (43)$$

The second term in the right-hand side of Eq. (43) is due to the $w(\delta\alpha)$ terms which distort the symmetric $\delta\alpha$ distribution.

Let us compare the obtained expressions for $\overline{S(Y, \delta\alpha)}$ with the results given in [3] for the same nonlinearity types.

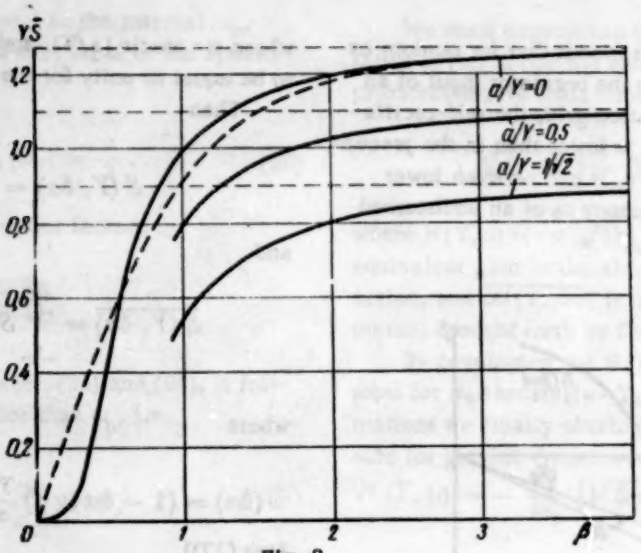

Fig. 3.

Figure 3 shows the $Y\overline{S(Y, \delta\alpha)}$ dependence plotted as a function of the $\beta = Y/\sqrt{2}\sigma$ parameter according to Eq. (42) and the curve, shown by the dotted line, plotted for the same case according to the equation given in [3]. The same diagram shows the $Y\overline{S(Y, \delta\alpha)}$ curves for $a/Y = 0.5$ and $a/Y = 1/\sqrt{2}$, which were obtained according to Eq. (43) without taking into account the distortion of the symmetry of $w(\delta\alpha)$. Similar dependences based on equations used in [3] are not provided here, since, for $\sigma \to 0$, one can use only a finite number of terms of the double infinite series, which expresses the equivalent gain value in this case in expanding the indeterminacies for their calculation.

A comparison of characteristics for a relay without an insensitivity zone (Fig. 3) shows that both static-linearization methods yield approximately equal results, at least in the region of small values (in correspondence with the assumptions of the method proposed). The expression $\overline{S(Y, \delta\alpha)}$ which was found here was given in unexpanded form and not in the form of an infinite series as in [3], thereby considerably reducing the amount of work involved in calculation. In conclusion, it should be noted that the statistical linearization method used in [3] cannot reveal the presence of a complex component in the expression for the equivalent gain for a relay with an insensitivity zone, which is important, for instance, in analyzing the stability of periodic regimes.

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EXTREMAL CONTROL BY THE METHOD OF RANDOM SCANNING

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The present article describes a method of random scanning in application to problems in extremal control of multiparameter systems. A device which realizes this method is described.

1. Formulation of the Problem

In the majority of cases, the problems encountered in the synthesis of self-adjusting systems, which are currently gaining widespread use, are reduced to the optimization of a certain performance function Q of these systems, i.e., to the search for conditions which secure the minimum value Q^* of this function.

Let Q be a function of the parameters $x_1, x_2, \ldots, x_n$,

$$Q = Q(x_1, x_2, \ldots, x_n), \tag{1}$$

while Q^* is the smallest of all possible values of this function:

$$Q^* = Q(x_1^*, \ldots, x_n^*) \leqslant Q(x_1, \ldots, x_n) \quad (-\infty < x_i < \infty,\ i = 1, \ldots, n). \tag{2}$$

It is necessary to find values of the $x_1^*, x_2^*, \ldots, x_n^*$ parameters for which the Q function is minimized, i.e., values which satisfy expression (2).

It is obvious that in order to solve this problem it is necessary to ascertain the existence of at least one minimum of the performance function. The number of minima is, however, not limited. It will be shown below that the proposed scanning system discriminates between the values of the minima and statistically tends to secure the smallest of these minima.

At present, there are several automatic optimization methods, i.e., methods of searching for the optimizing series of parameters $x_1^*, \ldots, x_n^*$. From among these methods, the method of alternating parameter changes (the Gauss-Zeidel method), the gradient method, and the method of quickest triggering (the Kantorovich method) should be mentioned. These methods have been applied and are presently used in a large number of automatic optimization systems (see, for instance, [1]).

The scanning organized in correspondence with the above methods has the character of predetermined scanning, i.e., of scanning where the changes in the $x_1, x_2, \ldots, x_n$ parameters are rigorously predetermined by the type of function Q and the organization of the selected scanning method.

However, predetermined scanning is not the only possible method. W. R. Ashby [2] came forward with his idea of random scanning, which was used as the basis of the present paper.

2. The Random Scanning Method

In terms of the control theory, Ashby's idea can be represented in the following manner.

Assume that the system to be controlled can exist in a certain multiplicity set $P = \{\bar{p}\}$ of states, finite or infinite. In this case, the system is considered to be adjusted if it is in one of the states contained in the multiplicity subset $P^* \subset P$. The only requirement imposed on this multiplicity subset is that it be of equal power to the multiplicity set P.

Thus, the problem of the adjustment of the system to be controlled is reduced to the selection of a method and organization of searching for one of the $\bar{p}^*$ elements of the multiplicity set P^* among all possible elements $\bar{p}$ in the multiplicity set P. Ashby suggested that $\bar{p}^*$ be sought not with respect to a given rule, as in the case of predetermined scanning, but by means of random selections to be performed by a "selection amplifier" [3].

Ashby's selection amplifier operates in the following manner (see Fig. 1). A certain state $\bar{p}$, selected at random, is communicated to the system to be controlled SC by means of a generator of random events, GRE, which secures a regime of random selections from the source of all possible states SS, i.e., from the multiplicity set P. In this, the selector unit SU, after receiving information on the reaction of the controlled system to

the variant supplied by GRE, either sends a command to GRE to produce the next random selection in the case of $\bar{p} \neq \bar{p}^*$ or, in the case of $\bar{p} = \bar{p}^*$, disconnects GRE and thereby locks the system in this state.

It is obvious that, in operation with respect to such commands, the selection unit will screen out the $\bar{p} \neq \bar{p}^*$ states until it finds, by chance, the required state, which it will preserve continuously until any external or internal factors transfer the system from this state into another $\bar{p} \neq \bar{p}^*$ state. After this, the above-considered mode of random searching for the state $\bar{p}^*$ will again become operative. W. R. Ashby's well-known homeostat operates on the basis of this principle [2 and 3].

It should be noted that Ashby's system is characterized by three specific features: it has a source of all possibilities (the multiplicity set P of elements), a device securing a regime of random selections from a general totality P (generator of random events), and a device which supplies the criterion according to which the required element is selected in the system to be controlled (selection unit). Another important feature is the presence of two sharply different operating conditions: the temporary, transient random scanning mode (for $\bar{p} \neq \bar{p}^*$) and the other, the stable adjusted mode (for $\bar{p} \neq \bar{p}^*$), which was to be found. It can readily be seen the the described method is essentially the realization of the trial and error method, which is widely used in practice.

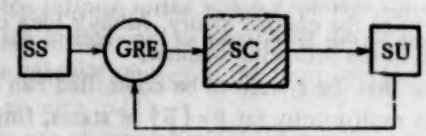

Fig. 1.

3. Application of Random Scanning to Optimization Problems

In order to apply the described random scanning to the problem of minimizing the $Q(x_1, x_2, \ldots, x_n)$ function, the "state" $\bar{p}$ of the "system to be controlled" Q must denote the velocities of its motion along the coordinates, i.e., $\bar{p} = (\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n)$, rather than its coordinates in a parametric coordinate system. It is obvious that $\bar{p}$ is the velocity vector of a point representing the state of the system to be controlled in an n-dimensional parametric space $\{x_1, x_2, \ldots, x_n\}$. Under these conditions, the control system depicted in Fig. 1 is applicable. The above-mentioned operating modes of the system are also preserved, with the exception that the system may not be immediately transferred to the Q^* state under the adjusted conditions ($\bar{p} = \bar{p}^*$) for $n > 1$; however, in this case, it will "approach" the required state, i.e., the performance function Q will have a smaller value at the end of the adjustment conditions duration than at the beginning.

As the most natural criterion for the selection of states supplied at random, we can use, for instance, the value of the derivative

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial x_1}\dot{x}_1 + \ldots + \frac{\partial Q}{\partial x_n}\dot{x}_n. \qquad (3)$$

If all possible states $\bar{p} = (\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n)$ which the state source SS can supply pertain to the multiplicity set P, the multiplicity set P^* must contain those states of the system to be controlled which satisfy the inequality

$$\frac{dQ}{dt} < 0, \qquad (4)$$

i.e., the condition for the diminution of the performance function. This inequality represents the selection criterion. It can readily be seen that the probability that the velocity $\bar{p}$ satisfying this criterion will be selected as close to $\frac{1}{2}$, since the "quantity" $\bar{p}^* \in P^*$ is slightly less than one half of all $\bar{p} \in P$ if the hypersurfaces $Q(x_1, x_2, \ldots, x_n) = \text{const}$ in an n-dimensional parametric space are sufficiently smooth.

Thus, the algorithm of random scanning in the given case consists in the fact that the $\bar{p}$ states, which lead to $(dQ/dt) \geq 0$, are transient and are immediately replaced by other states also chosen at random. The states leading to the inequality $(dQ/dt) < 0$ are stabilized until this inequality changes its sign due to the change in the derivative. It is obvious from (3) that the change in the derivative in the fixed state, i.e., for $\bar{p} = \text{const}$, occurs due to changes in partial derivatives $(\partial Q/\partial x_i)$ $(i = 1, 2, \ldots, n)$ during the motion of the point representing the system to be controlled in the parametric space. The transition from the adjustment value of $\bar{p}$ to another value takes place at the moment at which the derivative changes its sign, i.e., when $(dQ/dt) = 0$. It can readily be seen that, as a result of operation with respect to the indicated algorithm, the quantity Q can be reduced to its minimum value Q^* after a certain finite number of cycles in selecting $\bar{p}$.

However, the realization of such an algorithm requires the calculation of the (dQ/dt) derivative in correspondence with the selection criterion (4), which, in the end reduces the noise stability of this control system, especially for small values of the derivative modulus. Therefore, we chose a different method that did not require the calculation of the derivative and was simple and reliable, although it did reduce the response quickness in adjustment to a certain extent.

We proposed an arrangement whereby the Q value of the performance function is continuously compared with a certain level η, which changes at a constant rate $q < 0$. Let the values of Q and η coincide at the initial moment.

Then, $\eta(t) = Q_i + qt$, where Q_i is the value of the Q function at the beginning of the ith stage, and t is the time.

Let us denote by $Q(\bar{p}, t)$ the performance function value at the instant of time t for the fixed rate $\bar{p}$ of changes in the system parameters. If, for the rate value $\bar{p}_i$ at the beginning of the ith stage and for sufficiently small values of $t > 0$ the inequality

$$Q(\bar{p}_i, t) - \eta(t) < 0, \qquad (5)$$

holds, the Q_{i+1} value of the performance function at the instant of time $t = t_i$, when the $\eta(t)$ and $Q(\bar{p}_i, t)$ values are compared again, is smaller than the Q_i value at the initial moment. At the instant of time t_i, the rate value is switched from $\bar{p}_i$ to the $\bar{p}_{i+1}$ value, and a new stage in the process is initiated.

Consequently, if inequality (5) is satisfied after the beginning of the ith stage, the performance function value is smaller at the beginning of the next stage.

Thus, inequality (5) can serve as the selection criterion. Then, all values of $\bar{p}$ satisfying this inequality pertain to the multiplicity subset P^*. The $\bar{p}$ values that do not satisfy (5) are immediately rejected.

Criterion (5) does not require the calculation of the derivative, thus considerably simplifying its realization; however, it is somewhat narrower than criterion (4). Actually, inequality (5) for $t = 0$ can be written in the differential form:

$$\frac{dQ}{dt} < q. \qquad (6)$$

The obtained inequality determines the selection of rate p at the initial moment. The smaller the q value, the smaller the domain of $\bar{p}$ values which satisfy this inequality. Therefore, by comparing (4) and (6), we ascertain that since $q < 0$, the domain of rate values satisfying (5) is always smaller than the domain of rate values satisfying (4).

Figure 2 shows the variation of Q for two values of $\bar{p}_i$. The thin lines in the same figure indicate η levels for different values $q_1 > q_2 > q_3$. It can readily be seen that, for $q = q_3$, the $\bar{p}_i^{(2)}$ value does not belong to P^*, while $\bar{p}_i^{(1)} \in P^*$ in the T_i interval. With an increase in q, the multiplicity set P^* becomes wider. Thus, for $q = q_1$, both rate values $p_i^{(1,2)} \in P^*$.

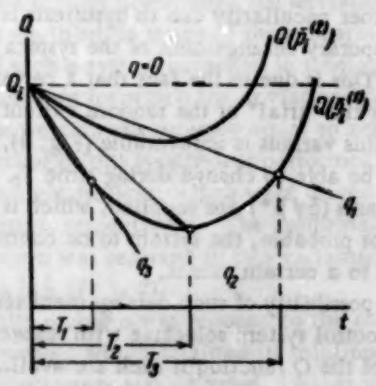

Fig. 2.

Therefore, we can consider that

$$P^* = P^*(q) \qquad (7)$$

has been established.

For $q = 0$, inequalities (4) and (6) coincide, and the selection of rates at the initial moment will be equal for both criteria. However, at the switching moment, criterion (6) will yield $Q_{i+1} = Q_i$, i.e., there will be no adjustment. For large $q < 0$, the adjustment will be weak, since it follows from (5) that the $Q_i - Q_{i+1} = -qt$ difference is small, regardless of the fact that the selected rate led to $Q < Q_{i+1}$ values during the operation process (see the $q = q_3$ case for $\bar{p}_i^{(1)}$ in Fig. 2).

Thus, on the one hand, an increase in q leads to the enlargement of the multiplicity subset P^* and, on the other hand, lowers the adjustment efficiency. Therefore, there exists such a $q = q^*$ value which optimizes the considered control system with respect to adjustment speed.

The η level is essentially one of the memory organization forms where $\eta > 0$ is "memorized" instead of Q. In this case, the "memory" is shifted in time independently of the Q value until the moment at which $Q = \eta$. The presence of the $\eta - Q$ difference, the sign of which determines the device operating conditions, considerably improves its noise stability.

It should be noted that this memory form is applicable also in predetermined scanning, which is organized according to the method of alternating parameter changes or according to the quickest triggering method. In this case, there is no necessity of determining the minimum of the performance function Q in the selected direction with respect to criterion (4), and it is sufficient to introduce the described "memory" by means of the comparison signal. Such an improvement, as was shown above, considerably improves the control system noise stability, although it somewhat lowers the scanning rate.

Figure 3 shows the block diagram of a device capable of realizing extremal control by the described method of random scanning.

In this diagram, the GRE generators secure a regime of random selections from the SS source of all possible states and act on the rate of change in the parameters of the system to be controlled, SC. In this, the changing quantity Q is continuously compared with a certain signal η generated by the generator of comparison signals, GCS. The result of the $Q \gtrless \eta$ comparison determines the system operating mode, which is assigned by the control unit, CU. The operating modes are given in the table.

TABLE 1.

Operating modes	C	Q	η
preparation	$Q>\eta$	const	$c>0$
scanning	$Q<\eta$	var	$q<0$

During the preparation stage, GRE's are disconnected, and the controlled system is not excited, i.e., the tracing point is immobile and Q = const. For $Q=\eta$, the preparation operating mode is replaced by the scanning mode and during this stage the system parameters change at the constant rate $\bar{p}$, the direction and modulus of which are determined by GRE at random. At the moment when $Q=\eta$, the scanning mode is replaced by the preparation mode, etc.

It can readily be shown that the indicated device corresponds to the above-considered algorithm of random scanning with the selection criterion (5). Here, the q-parameter has a rigorously defined physical meaning: it is the rate of change in the comparison signal η under scanning conditions. It is obvious that, in the absence of hysteresis in the considered control system, the operation is possible only for

$$\dot{Q}_{\min} < q. \qquad (8)$$

In the opposite case, the $Q<\eta$ inequality would never be satisfied.

4. Effects of Hysteresis

It has been shown that the described device operates under two regimes: "scanning" and "preparation," and that the moment of transition from one regime to another is determined by the coincidence of Q and η signals. However, in actual devices, due to a number of causes (inertia, hysteresis of the comparison system, etc.), the switching of regimes takes place for different relations between η and Q. Thus, the moment of transition from the preparation regime to the scanning regime or the reverse is determined by the equations,

$$\eta = Q + \delta_1, \quad \eta = Q - \delta_2,$$

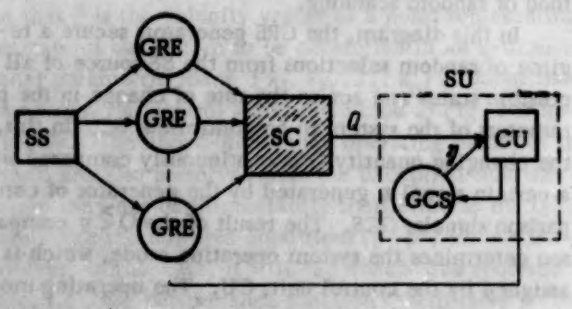

Fig. 3.

respectively, where δ_1 and δ_2 are usually sufficiently small positive quantities: $\delta_1=\delta_1(c)$ and $\delta_2=\delta_2(q, \bar{p})$.

It can readily be seen that, for disconnected GRE, i.e., for Q = const, self-oscillation conditions will arise —sawtooth oscillations of the η signal with respect to Q, where $\eta_{\max}-\eta_{\min}=\delta_1+\delta_2$.

These self-oscillations, a consequence of hysteresis in actual control systems, introduce interesting peculiarities in the device operation.

Thus, in the presence of self-oscillations, control is possible even if inequality (8) is not satisfied, i.e., if $|\bar{p}|_{\max} < |q|$. This is obvious from the graph in Fig. 4,

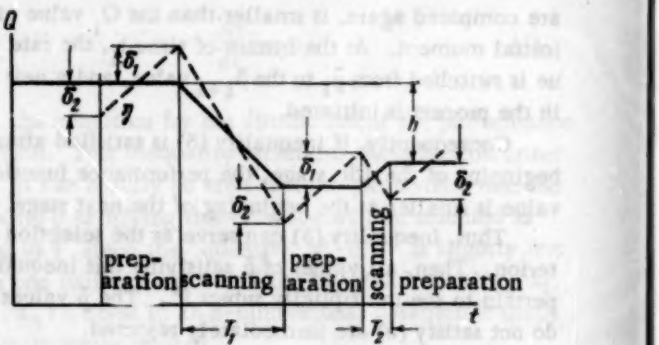

Fig. 4.

where two scanning cycles for the favorable (T_1 interval) and the unfavorable (T_2 interval) variants of the Q function variation (solid line) are shown. As a result of the action of both variants, the peformance function is reduced by h due to the fact that, in adjustment, the mutual rate of change in Q and η is less than in the unadjusted state. This leads to a change in the duration of the above regimes: $T_1 > T_2$.

Thus, as a result of hysteresis for a large $|q|$, the η signal follows the Q quantity under self-oscillation conditions, while the duration of favorable variants of Q variation ($\dot{Q}<0$) is somewhat longer than that of unfavorable variants ($\dot{Q}>0$) which ultimately leads to a systematic reduction of the performance function. Therefore, in certain cases, the deliberate introduction of hysteresis into the control system is entirely possible.

Another peculiarity due to hysteresis is the possibility of temporary derangement of the system to be controlled. This is due to the fact that a certain time is required for the "trial" of the random variant offered by a GRE. If this variant is unfavorable (Fig. 4), the Q function will be able to change during time T_2. If several such variants ($\bar{p}\neq\bar{p}^*$) are supplied, which is possible although not probable, the system to be controlled will be deranged to a certain extent.

The possibility of such derangement renders the described control system selective with respect to different minima of the Q function if such are available. Actually, after Q attained the intermediate minimum B (Fig.

5), the system will continue its motion in the neighborhood of the point B until the system, in its random motion due to the possibility of derangement, reaches the point C. From this point, the system can either return in the opposite direction or, due to the minimizing action of the described device, reach the neighborhood of the point A. In the first case, the whole process will be repeated from the beginning until the tracing point reaches point A – the smallest minimum of Q. Naturally, motion

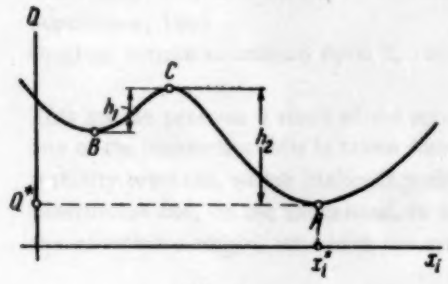

Fig. 5.

in the opposite direction, from A to B, is also possible; however, derangement variants which occur considerably less often are required in this case and, consequently, a longer time for overcoming the $h_2 > h_1$ barrier will be required. Consequently, the probability of a transition from A to B is smaller than the probability of transition from B to A. Thus, the system, being able to dwell in any region, will remain for a considerably longer time in the neighborhood of its minimum value Q^* than in any other region.

5. Experimental Verification

The possibility of optimization by the random scanning method according to the proposed scheme was checked experimentally under laboratory conditions in application to simple self-adjusting systems.

We considered the problem of the self-balancing of two independent small beam-weights which formed a system having two degrees of freedom. The beams were maintained in the horizontal position by means of leaf springs whose tension determined the deviations x_1 and x_2 from the equilibrium states of these beams. The adjustment of such a system consisted in finding the equilibrium conditions for which the deviations x_1 and x_2 of these beams were simultaneously equal to zero: $x_1 = x_2 = 0$. The adjustment of this system was performed by means of mobile weight trolleys, which were driven by two electric motors, respectively. The performance function Q of the system was realized in two variants: $Q = |x_1| + |x_2|$ and $Q = \max |x_i|$ $(i = 1, 2)$.

Thus, the problem was reduced to the minimization of the Q function, which attained its minimum ($Q^* = 0$) only when the system was adjusted, i.e., at the moment at which both beams were in equilibrium.

The comparison signal generator consisted of a voltage divider – a rheostat which was electromechanically driven. In the first case, the selection unit was based on a three-coil polarized RP-5 relay, where $|x_1|$ and $|x_2|$ were fed to two windings, and the comparison signal η was fed to the third winding. The second variant was based on two two-coil polarized relays. The random event generator consisted of an ordinary PS-64 scaling device, to the input of which noise from a tube was supplied; when the noise was disconnected, the trigger cells were fixed in a random position which determined the motion of the weight trolleys that balanced the beams.

The performed experiments showed that:

1. Adjustment, i.e., minimization of the Q function in both variants, is possible through use of the described method.

2. The adjustment itself is independent of the rate q of signal variation η in a wide range; for different values of q, only the average adjustment time changed.

3. There exists an optimum value $q = q^*$ for which the average time for the system adjustment is at the minimum, all other conditions being equal, i.e., there exists a q for which the described adjustment system is optimized with respect to quickness of response.

Another experiment was prepared for verifying the possibility of adjustment by the random scanning method in the presence of a systematic disturbance factor. For this purpose we used a beam with two trolleys, one of which moved at a constant speed, thereby introducing the disturbance, while the other was included in the above-described control system for the purpose of adjusting (balancing) this single-parameter system.

The performance function was realized in the natural way:

$$Q = |\dot{x}|,$$

where x is the beam deviation from the equilibrium position. Such a device is essentially the simplest statistical follow-up system operating on the basis of the random scanning method.

Experiments showed that the performance of such a self-adjusting system is largely determined by the process speed and, consequently, by the selection of the q parameter.

SUMMARY

The proposed extremal control system, based on random scanning, can be successfully applied in automatic optimization of multiparameter systems to be controlled for any performance function, including discontinuous functions. The advantages of this system are particularly pronounced in the case of control with respect to a large number of independent or dependent parameters of the system to be controlled. Computers, which are necessary in the case of predetermined scan-

ning, are not required in the realization of such a system, making it possible to optimize the system to be controlled with a minimum amount of equipment. The increased noise stability, secured by the application of memory in the form of reference signals, can also be used in predetermined extremal control systems.

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STABILITY OF A CONTROLLED SYSTEM WITH TWO NONLINEAR ELEMENTS

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This article presents a study of the second B. V. Bulgakov problem where the nonlinearity of one of the measuring units is taken into account and a method is described for solving the stability problem, which makes it possible, on the one hand, to find acceptable regulator parameters and, on the other hand, to determine the controlled variables range, containing the coordinate origin, for which the system stability is guaranteed.

1. Formulation of the Problem

In their paper, A. M. Letov and A. I. Lur'e [1] formulated one of the most important problems in automatic control: the determination of stability criteria for controlled systems described by equations of the following form:

$$\dot{\eta}_k = \sum_{\alpha=1}^{n} b_{k\alpha}\eta_\alpha + n_k\xi,$$
$$\dot{\xi} = f(\sigma), \quad \sigma = \sum p_\alpha(\eta_\alpha) - \varphi(\xi), \tag{1}$$

where $p_\alpha(\eta_\alpha)$ and $\varphi(\xi)$ are functions of the (A) class. This paper also treats the effect of various parameters characterizing nonlinear dependences on stability of systems to be controlled. The problem stated is of great practical importance. Actually, let us consider an aircraft controlled with respect to yawing by a nonlinear autopilot having rigid feedback. This autopilot has a damping gyroscope which can be described by the following equations:

$$T_s^2\ddot{\alpha} + U_s\dot{\alpha} + K_s\alpha = \varphi_s(\bar{\sigma}),$$
$$\bar{\sigma} = M\dot{\psi} + N^2\ddot{\psi},$$

where α is the deviation angle of the damping gyroscope and ψ is the aircraft yaw angle. The $\varphi_s(\bar{\sigma})$ function (see Fig. 1) is of the following form:

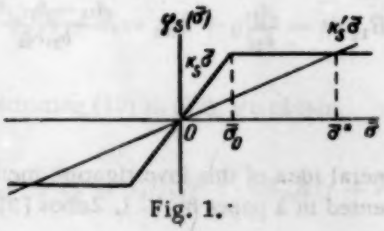

Fig. 1.

$$\varphi_s(\bar{\sigma}) = \begin{cases} k_s\bar{\sigma}_0 & \text{for } \bar{\sigma} \geqslant \bar{\sigma}_0, \\ k_s\bar{\sigma} & \text{for } |\bar{\sigma}| < \bar{\sigma}_0, \\ -k_s\bar{\sigma}_0 & \text{for } \bar{\sigma} \leqslant -\bar{\sigma}_0. \end{cases} \tag{2}$$

The T_s^2 and U_s coefficients are usually small and, in the case where the $\varphi_s(\bar{\sigma})$ function is linear, they can be neglected [2]. Then, the equations of the aircraft motion with respect to the yaw angle can be approximately represented in the following form:

$$T^2\ddot{\psi} + U\dot{\psi} + K\psi + \mu = 0,$$
$$\dot{\mu} = f^*(\sigma), \quad \sigma = \alpha\psi + E\alpha - \frac{1}{2}\mu, \tag{3}$$
$$\alpha = \varphi^*(\bar{\sigma}), \quad \bar{\sigma} = M\dot{\psi} + N^2\ddot{\psi}.$$

Here, ψ is the yaw angle, μ is the rudder deviation angle, $\varphi^*(\bar{\sigma}) = (1/K_s)\varphi_s(\bar{\sigma})$ and $f^*(\sigma)$ is the servomotor characteristic (see Fig. 2), which has the following properties:

$$\left[\frac{df^*(\sigma)}{d\sigma}\right]_{\sigma=0} \geqslant h_0^* > 0,$$
$$|h_1^*\sigma| \geqslant |f^*(\sigma)| \geqslant |h_2^*\sigma| \text{ for } \sigma \neq 0. \tag{4}$$

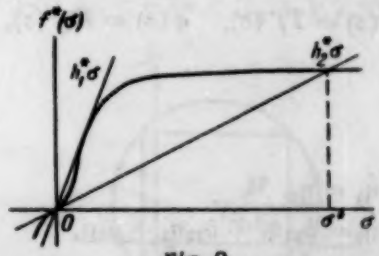

Fig. 2.

The problem of the stability of system (3) in the case where the $\varphi^*(\bar{\sigma})$ function is linear has been thoroughly investigated in [2]. The following question now

arises: will the stability of system (3) deteriorate if the $\varphi^*(\bar{\sigma})$ function is nonlinear or will it be possible to find such autopilot parameters for which this nonlinearity will not impair the stability of system (3) in comparison with the case wherein the gyroscope characteristic is linear. The purpose of the present paper is to determine the effect of the nonlinearity of $\varphi^*(\bar{\sigma})$ on the aircraft flight stability and to find the stability criteria for the system (3). It is obvious that system (3) is a particular case of system (1).

2. General Form of the Stability Conditions

In investigating the stability of system (3), we shall use the second (direct) Lyapunov method. The Lyapunov V function is usually plotted in quadratic form, and the coefficients in the expression for $\dot{V}$ do not depend on the system coordinates. However, as is known, the latter condition is not compulsory. It is sometimes difficult to plot the Lyapunov function in such a way that the coefficients of the $\dot{V}$ form are independent of the system coordinates. Actually, if some coefficients of the $\dot{V}$ form depend on the system coordinates, the problem consists in finding a continuous continuum of these coordinates, which contain the coordinate origin and which lie within a certain interval (in the control range), and in selecting the remaining coefficients in such a way that the V function is actually a Lyapunov function for the given system. This method has been chosen here for solving the above problem.*

Before we start plotting any diagrams, let us reduce the initial equations (3) to the normal and dimensionless forms.

By introducing the following notation:

$$\eta_1 = \psi, \quad \eta_2 = T\frac{d\psi}{dt}, \quad t = T\tau, \quad b_{21} = -K, \quad b_{22} = -\frac{U}{T},$$

$$u_2 = -1, \quad p_1 = a, \quad q_1 = -\frac{N^2K}{T^2}, \quad q_2 = \left(M - \frac{N^2U}{T^2}\right)\frac{1}{T}, \quad r = \frac{1}{l}, \quad \bar{r} = \frac{N^2}{T^2}, \quad f(\sigma) = Tf^*(\sigma), \quad \varphi(\bar{\sigma}) = E\varphi^*(\bar{\sigma}), \tag{5}$$

we obtain:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2, \\ \dot{\eta}_2 &= b_{21}\eta_1 + b_{22}\eta_2 + u_2\mu, \\ \dot{\mu} &= f(\sigma), \\ \sigma &= p_1\eta_1 + \varphi(\sigma) - r\mu, \\ \bar{\sigma} &= q_1\eta_1 + q_2\eta_2 - \bar{r}\mu. \end{aligned} \tag{6}$$

It is obvious from Fig. 2 that, in the control range $|\sigma| \leq |\sigma^*|$, the entire $f^*(\sigma)$ function is located between two straight lines $y = h_1^*\sigma$ and $y = h_2^*\sigma$, where $h_1^* > h_2^*$. In this interval, $h(\sigma)\sigma$ can be substituted for $f(\sigma)$, where $h(\sigma) = (1/\sigma)f(\sigma)$,

$$h_1 = Th_1^* \geqslant h(\sigma) \geqslant Th_2^* = h_2,$$

$$h(0) = \lim_{\sigma \to 0}\frac{1}{\sigma}f(\sigma) = h_0 = Th_0^*.$$

Similarly, in the $|\bar{\sigma}| \leq \bar{\sigma}^*$ range, $\varphi(\bar{\sigma})$ can be replaced by $k(\bar{\sigma})\bar{\sigma}$, where

$$k(\sigma) = \frac{\varphi(\bar{\sigma})}{\bar{\sigma}}, \quad k_1 = E\frac{k_s}{K_s} \geqslant k(\sigma) \geqslant \frac{E}{K_s}k_s' = k_2.$$

Thus, system (6) can be written in a different form:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2, \\ \dot{\eta}_2 &= b_{21}\eta_1 + b_{22}\eta_2 + u_2\mu, \\ \dot{\mu} &= h(\sigma)\left[(p_1 + q_1k(\bar{\sigma}))\eta_1 + q_2k(\bar{\sigma})\eta_2 - (r + \bar{r}k(\bar{\sigma}))\mu\right]. \end{aligned} \tag{7}$$

We assume that system (6) remains stable after the automatic device is switched off, i.e., that all roots of the $\lambda^2 - b_{22}\lambda - b_{21} = 0$ equation have negative real parts. According to the well-known Lyapunov theorem, it is always possible to find a positive function

$$F = \tfrac{1}{2}\left(B_{11}\eta_1^2 + 2B_{12}\eta_1\eta_2 + B_{22}^2\eta_2^2\right)$$

whose sign is determined and which satisfies the equation

$$\frac{\partial F}{\partial \eta_1}\eta_2 + \frac{\partial F}{\partial \eta_2}(b_{21}\eta_1 + b_{22}\eta_2) = \overline{W} = -(A_{11}\eta_1^2 + 2A_{12}\eta_1\eta_2 + A_{22}\eta_2^2), \tag{8}$$

where $\overline{W}$ is an assigned negative function whose sign is determined, i.e., the A_{11}, A_{12}, and A_{22} coefficients, which are not yet known, satisfy the conditions

$$A_{11} > 0, \quad A_{11}A_{22} - A_{12}^2 > 0. \tag{9}$$

From Eq. (8), the expressions for the B_{11}, B_{12}, and B_{22} coefficients can readily be found in terms of the A_{11}, A_{12}, and A_{22} coefficients:

$$B_{11} = \frac{1}{-b_{21}b_{22}}\left[(b_{21} - b_{22}^2)A_{11} + 2b_{21}b_{22}A_{12} - b_{21}^2A_{22}\right], \quad B_{12} = -\frac{A_{11}}{b_{21}}, \quad B_{22} = \frac{A_{11} - b_{21}A_{22}}{b_{21}b_{22}} \tag{10}$$

* The general idea of this investigation method has been presented in a paper by V. I. Zubov [3].

Let us now consider the following positive function whose sign is determined:

$$V = F(\eta_1, \eta_2) + \frac{1}{2}\mu^2.$$

According to Eqs. (7), we find:

$$-\dot{V} = -\overline{W} - u_2 B_{12}\eta_1\mu - u_2 B_{22}\eta_2\mu - h(\sigma)\left[p_1 + q_1 k(\bar{\sigma})\right]\eta_1\mu - q_2 h(\sigma) k(\bar{\sigma})\eta_2\mu + h(\sigma)\left[r + \bar{r}k(\bar{\sigma})\right]\mu^2.$$

According to the Sylvester criterion, $\dot{V}$ will be a negative function with determined sign if the following inequality is satisfied:

$$D = \begin{vmatrix} A_{11} & A_{12} & P_1 \\ A_{12} & A_{22} & P_2 \\ P_1 & P_2 & h(\sigma)\left[r + \bar{r}k(\bar{\sigma})\right] \end{vmatrix} > 0, \qquad (11)$$

where $P_1 = \frac{1}{2}\left[u_2 B_{12} + h(\sigma)\left(p_1 + q_1 k(\bar{\sigma})\right)\right]$,

$$P_2 = \frac{1}{2}\left[u_2 B_{22} + q_2 h(\sigma) k(\bar{\sigma})\right].$$

According to the well-known Lyapunov theorem, system (6) will be asymptotically stable if inequalities (9) and (11) are satisfied. Inequality (11) must be satisfied for continuous continua of the σ and $\bar{\sigma}$ coordinates which fill the $|\sigma| \le \sigma^*$ and $|\bar{\sigma}| \le \bar{\sigma}^*$ regions. It is obvious from Figs. 1 and 2 that the σ^* and $\bar{\sigma}^*$ values correspond to the lower branches of the straight lines $y = h_2^* \sigma$ and $\bar{y} = k_3 \bar{\sigma}$ of the nonlinear functions. Therefore, we shall henceforth consider only domains where $\underline{h}$ and $\underline{k}$ change. The extent of these domains will determine the character of the system stability. This extent obviously depends on the autopilot parameters.

3. Investigation of Stability Conditions

We shall now pass to the investigation of stability conditions for system (6). In the particular given case, we assume that $A_{12} = 0$ and $A_{11} = A_{22} > 0$ for the sake of simplicity. Inequality (9) will then always be satisfied. By expanding the determinant D, we obtain:

$$A_{22}^2 h(r + \bar{r}k) - \frac{A_{22}}{4}\left[u_2 B_{12} + h(p_1 + q_1 k)\right]^2 - \frac{A_{22}}{4}\left[u_2 B_{22} + q_2 h k\right]^2 > 0. \qquad (12)$$

We shall denote:

$$q_2 h k = x, \quad p_1 h + q_1 h k = y. \qquad (13)$$

By substituting (13) in (12), we obtain:

$$(x + u_2 B_{22})^2 + (y + u_2 B_{12})^2 < 4A_{22} h(r + \bar{r}k). \qquad (14)$$

We can readily find the expression

$$h(r + \bar{r}k) = \frac{r}{p_1} y + \left(\frac{\bar{r}}{q_2} - \frac{r q_1}{p_1 q_2}\right) x = Ay + Bx,$$

where

$$A = \frac{r}{p_1}, \quad B = \frac{\bar{r}p_1 - r q_1}{p_1 q_2}.$$

Inequality (14) can be written in the following form:

$$[x + (u_2 B_{22} - 2A_{22}B)]^2 + [y + (u_2 B_{12} - 2A_{22}A)]^2 < 4A_{22}^2(A^2 + B^2) - 4A_{22}B u_2 B_{22} - 4A_{22}A u_2 B_{12}. \qquad (15)$$

Let us substitute the expression (10) for B_{12} and B_{22} in (15) and represent it in the form:

$$(X - x_0)^2 + (Y - y_0)^2 < 4R^2, \qquad (16)$$

where

$$X = \frac{x}{A_{22}}, \quad Y = \frac{y}{A_{22}}, \quad x_0 = 2B - \frac{u_2(1 - b_{21})}{b_{21} b_{22}},$$

$$y_0 = 2A + \frac{u_2}{b_{21}}, \quad R^2 = A^2 + B^2 + C,$$

$$C = -u_2\left[\frac{(1 - b_{21})B}{b_{21} b_{22}} - \frac{A}{b_{21}}\right].$$

It is obvious that X and Y are not constant quantities in inequality (16).

It can readily be shown that, in order that inequality (16) be satisfied, it is sufficient that the following inequalities hold (see Fig. 3):

$$(x_0 - \sqrt{2}R) < X < (x_0 + \sqrt{2}R),$$

$$(y_0 - \sqrt{2}R) \leqslant Y \leqslant (y_0 + \sqrt{2}R),$$

or

$$(x_0 - \sqrt{2}R) A_{22} < q_2 h k < (x_0 + \sqrt{2}R) A_{22}, \qquad (17a)$$

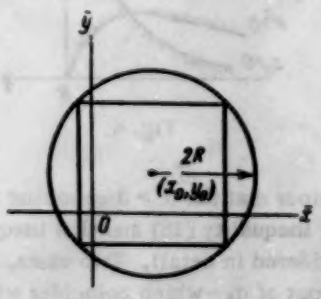

Fig. 3.

$$(y_0 - \sqrt{2}R)\,A_{22} \leqslant p_1 h + q_1 hk \leqslant (y_0 + \sqrt{2}R)\,A_{22}. \tag{17b}$$

In accordance with inequalities (17a) and (17b), we can find an allowable domain where $\underline{h}$ and $\underline{k}$ change and determine the effect of nonlinearity on the stability of system (6). Before this, according to the initial transformations of the coefficients, we find the expressions for R^2, x_0, and y_0:

$$R^2 = \frac{T^2(N^2al + N^2K)^2}{a^2l^2(MT^2 - N^2U)^2} + \frac{T^2(N^2al + N^2K)}{alKU(MT^2 - N^2U)} + \frac{al + K}{a^2l^2K},$$

$$x_0 = \frac{2T(N^2al + N^2K)}{al(MT^2 - N^2U)} + \frac{T(1+K)}{KU}, \quad y_0 = \frac{2}{al} + \frac{1}{K}.$$

Denoting

$$\eta = \frac{T(N^2al + N^2K)}{MT^2 - N^2U}, \quad \xi = \frac{K}{al}, \quad S = \frac{(1+K)T}{U}. \tag{18}$$

Equations (18) include the autopilot coefficients. In the given case, it is obvious that $\xi \geq 0$ and $s \geq 0$, and the expressions for R^2, x_0, and y_0 assume the following form:

$$R^2 = \frac{1}{K^2}(\xi^2\eta^2 + S\xi\eta + \xi^2 + \xi),$$
$$x_0 = \frac{1}{K}(2\xi\eta + S), \quad y_0 = \frac{1}{K}(1 + 2\xi). \tag{19}$$

Let us find the conditions under which the inequality (12) or (16) is satisfied. According to inequality (12), it is necessary that the $h \geq m > 0$ relation holds where $\underline{m}$ is a certain finite number. Due to property (4) of the $f(\sigma)$ function, the latter inequality can always be satisfied. The $R^2 > 0$ condition, which is necessary for inequality (16) to be positive, follows from inequality (16). Let us plot the $R^2 = 0$ graphs in the (η, ξ) plane and separate the domain where $R^2 > 0$ (Fig. 4).

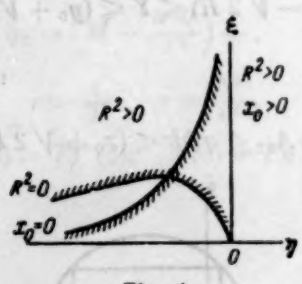

Fig. 4.

It is obvious that the $R^2 > 0$ condition is not sufficient for satisfying inequality (16) and that inequalities (17) must be considered in detail. Two cases, determined by the sign in front of q_2 —which coincides with the sign in front of η— are possible here.

1. In the case where $q_2 = (1/T^3)(MT^2 - N^2U) \geq 0$, inequality (17a) yields the condition:

$$\frac{x_0 + \sqrt{2}R}{x_0 - \sqrt{2}R} > \frac{h_1 k_1}{h_2 k_2} = \alpha\beta \geqslant 1, \tag{20}$$

where the quantities $\alpha = (h_1/h_2)$ and $\beta = (k_1/k_2)$ characterize the domain where $\underline{h}$ and $\underline{k}$ change.

Let us write inequality (20) in the form

$$\rho x_0 - \sqrt{2}R < 0, \tag{21}$$

where $\rho = \frac{\alpha\beta - 1}{\alpha\beta + 1} \leqslant 1$.

This inequality determines certain stability regions in the (η, ξ) plane of the autopilot parameters. The equation for the boundary of this region is $\rho x_0 - \sqrt{2}R = 0$. After substituting expression (19), this equation assumes the following form:

$$(4\rho^2 - 2)\xi^2\eta^2 + (4\rho^2 - 2)S\xi\eta - 2\xi^2 - 2\xi + \rho^2 S^2 = 0. \tag{22}$$

The boundaries of the stability region for different values of ρ are shown in the right (η, ξ) half-plane in Fig. 5.

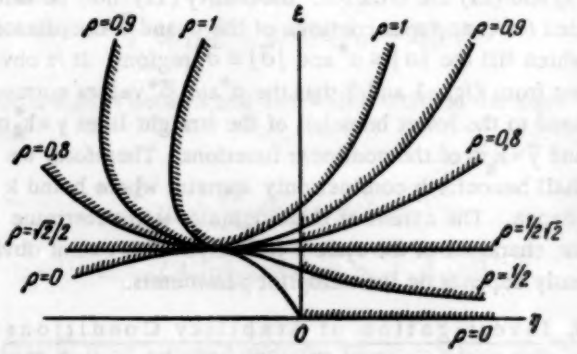

Fig. 5.

It is obvious that inequality (17a) can determine only the domain of the $\underline{h}$ and $\underline{k}$ quantities. Their maximum values are determined by inequality (17b).

Actually, inequality (17b) yields the conditions

$$\left[(y_0 - \sqrt{2}R) - \frac{q_1}{q_2}(x_0 - \sqrt{2}R)\right]\frac{A_{22}}{p_1} \leqslant$$
$$\leqslant h \leqslant \left[y_0 + \sqrt{2}R) - \frac{q_1}{q_2}(x_0 + \sqrt{2}R)\right]\frac{A_{22}}{p_1},$$

$$\frac{p_1(x_0 - \sqrt{2}R)}{q_2(y_0 - \sqrt{2}R) - q_1(x_0 - \sqrt{2}R)} \leqslant k \leqslant$$
$$\leqslant \frac{p_1(x_0 + \sqrt{2}R)}{q_2(y_0 + \sqrt{2}R) - q_1(x_0 + \sqrt{2}R)}.$$

The values of A_{22} and $\underline{k}$ have not yet been determined; suitable values of these quantities must be selected in order that the last two inequalities are satisfied.

2. In the case where $q_2 < 0$, $x_0(\eta, \xi)$ in the left (η, ξ) half-plane has different signs for different values of η and ξ, as is shown in Fig. 4. We have two cases, respectively:

for $x_0 > 0$, inequality (17a) yields the condition

$$x_0 - \sqrt{2}R < 0, \tag{23}$$

for $x_0 < 0$, inequality (17a) yields the condition

$$-\rho x_0 - \sqrt{2}R < 0, \tag{24}$$

where $\rho = \dfrac{\alpha\beta - 1}{\alpha\beta + 1} \leqslant 1$.

It can readily be shown that the equation for the boundary of the stability region, determined by inequalities (24), is of the same form as equation (22). The boundaries of the stability region for different values of ρ are shown in the left (η, ξ) half-plane in Fig. 5.

As in the preceding case, the value of $\underline{k}$ will be similarly determined by the inequality

$$\frac{(x_0 + \sqrt{2}R)\,p_1}{q_2(y_0 - \sqrt{2}R) - q_1(x_0 + \sqrt{2}R)} \leqslant k \leqslant$$

$$\leqslant \frac{(x_0 - \sqrt{2}R)\,p_1}{q_2(y_0 + \sqrt{2}R) - q_1(x_0 - \sqrt{2}R)}.$$

It is obvious from Fig. 5 that the stability region with respect to the η and ξ parameters is restricted as the value of ρ increases, i.e., as the domain wherein $\underline{h}$ and $\underline{k}$ change becomes larger. Therefore, generally speaking, the introduction of nonlinearity $\varphi(\bar{\sigma})$, which characterizes the gyroscope nonlinear property, reduces the stability of system (6). However, if, for a certain nonlinear function $\varphi(\bar{\sigma})$, corresponding, for instance, to $\rho = \rho_1$, $0 \leq \rho_1 \leq 1$, we choose the η and ξ parameters to be in the $\rho = \rho_1$ region, the nonlinearity will not affect the system stability. In this, in the region corresponding to $\rho = 1$, we can make the domain of $\underline{k}$ variations for which the system stability is guaranteed as large as we wish.

In conclusion, the author extends his thanks to A. M. Letov for his valuable advice and remarks.

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ON THE ACCURATE DETERMINATION OF PERIODIC REGIMES IN SECTIONALLY LINEAR AUTOMATIC CONTROL SYSTEMS*

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An exact method for determining periodic regimes in arbitrary sectionally linear systems is considered. The solution of the problem is reduced to the determination of forced oscillations of a linear system with periodically changing parameters. These forced oscillations are found in an exact manner by means of a new approach, which is based on the solution of an auxiliary Fredholm integral equation of the second kind. The thus obtained periodic solution depends on the unknown times of motion along sections of nonlinear characteristics. By using this solution and the conditions for the transition from one section of the nonlinear characteristic to another, it is possible to determine the transcendental equations of periods.

1. Formulation of the Problem

By a sectionally linear control system we shall designate a system which, for certain given time intervals, is described by different systems of linear differential equations with constant or variable coefficients. In the first case, such a system will be called a steady-state system, and in the second case, it will be called an unsteady-state system.

The simplest example of sectionally linear systems is those which differ from the linear systems by the presence of nonlinear elements having sectionally linear characteristics. Various spontaneously changing cybernetic systems can be considered as the more complex of this class [1]. Until now, only steady-state sectionally linear systems were generally considered in the literature [2-5]. This problem was first studied in [2] in which a method was presented for the determination of periodic regimes by fitting the linear-system solutions corresponding to different time intervals. In so doing, the solution of the problem was always brought to the point at which a certain transcendental algebraic system of equations was composed with respect to unknown times of motion for different sections of the nonlinear characteristic ("equations of periods").

The practical application of the fitting method in the form proposed in [2] often involves difficulties due to the following reasons:

1. The period equations depend on the roots of characteristic equations of the corresponding linear systems which, if the considered system of differential equations is of a high order, precludes the possibility of expressing the obtained results in terms of initial parameters.

2. The amount of calculation involved in composition of the period equations increases markedly with an increase in the order of the initial system of differential equations.

3. This method does not make it possible to take into account the filtering properties of the linear portion of the system and to determine only a finite number of harmonics in the Fourier expansion of the periodic regime to be found.

A modified fitting method for systems with a single nonlinearity, making it posssible to write the equations of periods in terms of the initial system coefficients, was proposed in [4]. In [3], a method different from those described in [2] and [4] was used in composing the equations of periods by fitting for the case where the system contained only one nonlinearity, the characteristic of which consisted of straight lines parallel to two given lines. The method used in [3] was based on a special affine transformation of the nonlinear characteristic and could not be extended to nonlinear characteristics of a more general type.

A linear transformation making it possible to reduce the amount of calculation involved in the application of the above methods was proposed in [5].

* The basic results of the present paper were reported at the Seminar on Nonlinear Problems in Automatic Control Theory, held at the Institute of Automation and Remote Control, Academy of Sciences, USSR, on December 2, 1959.

In the present paper, the solution of this problem is reduced to the consideration of the Fredholm integral equation of the second kind, providing the possibility of composing the equations of periods for an arbitrary sectionally linear system. An approximate method, making it possible to use only a finite number of harmonics of the periodic regime to be found, is also described. In this, all results are expressed directly in terms of the coefficients of the initial system of differential equations.

2. Transmission of Periodic Signals through the Linear Portion of the System

Consider the differential equations

$$\dot{x}_k = \sum_{\alpha=1}^{n} b_{k\alpha} x_\alpha + F_k(t) \quad (k = 1, 2, \ldots, n), \quad (1)$$

where $b_{k\alpha}$ are constants, and $F_k(t)$ are periodic functions of the period T, which we shall consider to be sectionally continuous. It is necessary to find the periodic solutions of the system (1). This problem has been studied in detail in [6] and [7]. Another solution form, necessary for further derivations, will be given below.

Let

$$D(\lambda) = |b_{k\alpha} - \lambda\delta_{k\alpha}| \quad (2)$$

be the characteristic determinant of the system linear portion,† and let $\Delta_{k\alpha}$ be the cofactor of the element of determinant (2) which is in the kth row and in the αth column. Subsequently, it is sufficient to assume that the equation

$$D(\lambda) = 0, \quad (3)$$

the roots of which are $\lambda = \lambda_\rho$ ($\rho = 1, 2, \ldots, n$), does not have a root equal to

$$\psi_s = \frac{2\pi i}{T} s, \quad (4)$$

where s is zero or any integer.

Under the above assumptions, the periodic solution of system (1) is uniquely defined by the equation

$$x_s^T(t) = \sum_{k=1}^{n} \int_0^T \Phi_{ks}^T(t-\tau) F_k(\tau)\, d\tau \quad (s = 1, 2, \ldots, n), \quad (5)$$

where

$$\Phi_{ks}^T(y) = -\frac{1}{T} \sum_{m=-\infty}^{\infty} \frac{\Delta_{ks}(\psi_m)}{D(\psi_m)} e^{\psi_m y} \quad (6)$$

Functions (6) can be represented in finite form. If $\lambda_\rho \neq \lambda_\alpha$ and for $\rho \neq \alpha$, we have

$$\Phi_{ks}^T(y) = \Phi_{ks}(y) = \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \frac{e^{\lambda_\rho y}}{e^{\lambda_\rho T} - 1}. \quad (7)$$

For y = 0 and y = T, the sum of the series (6) is equal to

$$\Phi_{kr}^T(0) = \Phi_{ks}^T(T) = \frac{1}{2} \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \operatorname{cth} \frac{\lambda_\rho T}{2} \quad \text{for} \quad 0 < y < T. \quad (8)$$

Moreover,

$$\Phi_{ks}(0) - \Phi_{ks}(T) = \delta_{ks}. \quad (9)$$

Consequently, for $k \neq s$, the $\Phi_{ks}^T(t-\tau)$ function is continuous with the square given by

$$0 \leqslant t \leqslant T, \quad 0 \leqslant \tau \leqslant T_1, \quad (10)$$

and the $\Phi_{ss}^T(t-\tau)$ functions have a discontinuity on the $t = \tau$ diagonal.

In using expression (7) in (5), it must be taken into account that, for $0 \leq t \leq T$, the argument $t-\tau$ of the integrand in (5) varies within the $-T \leq t-\tau \leq T$ interval and, in correspondence with (7),

$$\Phi_{ks}^T(t-\tau) = \begin{cases} \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \frac{e^{\lambda_\rho (t-\tau)}}{e^{\lambda_\rho T} - 1} & \text{for} \quad 0 < t-\tau < T, \\ \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \frac{e^{\lambda_\rho T} e^{\lambda_\rho (\tau - t)}}{e^{\lambda_\rho T} - 1} & \text{for} \quad 0 < \tau - t < T. \end{cases} \quad (11)$$

If Eq. (3) has the roots $\lambda_1, \ldots, \lambda_m$ (the multiplicities of which are $\nu_1, \nu_2, \ldots, \nu_m$), we shall have the following equation instead of (7):

$$\Phi_{ks}^T(y) = \sum_{\nu=1}^{\nu_m} \sum_{\rho=1}^{m} c_{\nu,\rho}^{ks} \frac{(-1)^\nu}{(\nu-1)!} \frac{d^{\nu-1}}{(d\lambda_\rho)^{\nu-1}} \left(\frac{e^{\lambda_\rho y}}{e^{\lambda_\rho T} - 1} \right) (0 < y < T), \quad (12)$$

where the $c_{\nu,\rho}^{ks}$ quantities are coefficients in resolution into partial fractions:

$$\frac{\Delta_{ks}(\lambda)}{D(\lambda)} = \sum_{\nu=1}^{\nu_m} \sum_{\rho=1}^{m} \frac{c_{\nu,\rho}^{ks}}{(\lambda - \lambda_\rho)^\nu}. \quad (13)$$

Also of interest are cases of $F_k(t)$ functions with the period T as well as functions

$$F_k(t) = -F_k(t+T). \quad (14)$$

Under the above assumptions, in case (14), the periodic solution of system (1) will be written thus:

$$x_s^T(t) = \sum_{k=1}^{n} \int_0^T \Psi_{ks}^T(t-\tau) F_k(\tau)\, d\tau, \quad (15)$$

where

$$\Psi_{ks}^T(y) = -\frac{2}{T} \sum_{m=-\infty}^{\infty} \frac{\Delta_{ks}(\psi_{2m-1})}{D(\psi_{2m-1})} e^{\psi_{2m-1} y}. \quad (16)$$

†$\delta_{k\alpha}$ is the Kronecker symbol.

If Eq. (3) has simple roots, we also have

$$\Psi_{ks}^{T}(y) =$$

$$= \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \frac{e^{\lambda_\rho y}}{e^{\frac{\lambda_\rho T}{2}} + 1} \quad \text{for } 0 < y < \frac{T}{2},$$

$$\Psi_{ks}^{T}(y) = -\Psi_{ks}^{T}\left(y + \frac{T}{2}\right). \tag{17}$$

The case wherein Eq. (3) has multiple roots is considered in the same manner as above.

The derivation of equations contained in this paragraph is given in Appendix I.

3. Self-Oscillations of a Steady-State System Having a Single Nonlinearity

Consider the system of equations

$$\dot{x}_k = \sum_{\alpha=1}^{n} b_{k\alpha} x_\alpha + h_k f(\sigma) \qquad (k = 1, 2, \ldots, n),$$

$$\sigma = \sum_{s=1}^{n} j_s x_s, \tag{18}$$

where $b_{k\alpha}$ are the same as in (1), h_k and j_s are constants, and $f(\sigma)$ is a sectionally linear function.

We shall introduce the notation

$$H_k(\lambda) = \sum_{s=1}^{n} h_s \Delta_{sk}(\lambda), \tag{19}$$

$$M(\lambda) = \sum_{k=1}^{n} j_k H_k(\lambda) = \sum_{s=1}^{n} \sum_{k=1}^{n} h_s j_k \Delta_{sk}(\lambda). \tag{20}$$

We shall asume that the polynomials (20) and (3) do not have common roots. Then without restricting the generality of treatment, we can consider that, for any number T, Eq. (3) does not have roots of the form (4), which can always be secured by introducing a new nonlinear function:

$$f_1(\sigma) = f(\sigma) - c\sigma,$$

where c is a certain given number.

We shall assume that system (18) has a periodic solution of a certain period T:

$$x_s^T(t) = x_s^T(t + T) \qquad (s = 1, 2, \ldots, n),$$

$$\sigma^T(t) = \sigma^T(t + T). \tag{21}$$

Then

$$f(t) = f(\sigma^T(t)) \tag{22}$$

will also be a periodic function of the T period.

Consider the linear system

$$\dot{x}_k = \sum_{\alpha=1}^{n} b_{k\alpha} x_\alpha + h_k f(t). \tag{23}$$

According to the above assumptions, we have a nonresonance case. In this case, the unique periodic solution of system (23) will obviously coincide with the solution (21) of system (18).

Since in the given case

$$F_k(t) = h_k f(t), \tag{24}$$

then, according to (5),

$$x_s^T(t) = \int_0^T \Phi_s^T(t - \tau) f(\tau)\, d\tau, \tag{25}$$

where

$$\Phi_s^T(y) = -\frac{1}{T} \sum_{m=-\infty}^{\infty} \frac{H_s(\psi_m)}{D(\psi_m)} e^{\psi_m y}, \tag{26}$$

and, moreover,

$$\sigma^T(t) = \int_0^T \Phi^T(t - \tau) f(\tau)\, d\tau, \tag{27}$$

where

$$\Phi^T(y) = \sum_{s=1}^{n} j_s \Phi_s^T(y) = \frac{1}{T} \sum_{m=-\infty}^{\infty} \frac{M(\psi_m)}{D(\psi_m)} e^{\psi_m y}. \tag{28}$$

Let us now consider the function (22). We shall assume that the periodic solution type (i.e., the sequence of passage over different portions of the nonlinear characteristic) is given. Then, we have

$$f(t) = a_i \sigma^T(t) + b_i \quad \text{for } t_{i-1} < t < t_i$$

$$(i = 1, 2, \ldots, m), \tag{29}$$

where a_i and b_i are assigned numbers, some of which can be equal to each other; m is the over-all number of transitions from one portion of the nonlinear characteristic to another during a single period; $t_0 = 0$;

$$0 < t_1 < t_2 < \ldots < t_m = T \tag{30}$$

are unknown times which must still be determined.

Let us introduce the periodic step functions

$$A(t) = A(t+T), \quad A(t) = a_i \text{ for } t_{i-1} < t < t_i,$$

$$B(t) = B(t+T), \quad B(t) = b_i \text{ for } t_{i-1} < t < t_i. \tag{31}$$

Then, (29) can be rewritten thus:

$$f(t) = A(t)\sigma^T(t) + B(t). \tag{32}$$

By substituting this expression in (27), we obtain the integral equation

$$\sigma^T(t) =$$

$$= \int_0^T \Phi^T(t-\tau) A(\tau) \sigma^T(\tau) d\tau + \int_0^T \Phi^T(t-\tau) B(\tau) d\tau,$$

whose kernel

$$R_1(t, \tau) = \Phi^T(t-\tau) A(\tau) \tag{34}$$

and the free term

$$M(t) = \int_0^T \Phi^T(t-\tau) B(\tau) d\tau \tag{35}$$

are known functions of t_i.

We shall limit our consideration to values of t_i for which the denominator of the resolvent of kernel (34) does not become equal to zero. Then, in correspondence with the general theory [8], we can write

$$\sigma^T(t, t_1, t_2, \ldots, T) = M(t) + \int_0^T R(t, \tau) M(\tau) d\tau, \tag{36}$$

where $R(t,\tau)$ is the resolvent of kernel (34).

If

$$|R_1(t, \tau)| < \frac{1}{T},$$

the resolvent can be obtained by the method of successive approximations.

In the general case [8], the resolvent is given by the equation

$$R(t, \tau) = \frac{\sum_{n=0}^{\infty} (-1)^n \frac{d_n(t, \tau)}{n!}}{\sum_{n=0}^{\infty} (-1)^n \frac{d_n}{n!}}, \tag{37}$$

while

$$d_0(t, \tau) = R_1(t, \tau), \quad d_0 = 1, \tag{38}$$

and all remaining $d_n(t, \tau)$ and d_n can be successively found from the recurrent relations

$$d_{k+1} = \int_0^T d_k(\tau, \tau) d\tau, \tag{39}$$

$$d_n(t, \tau) = R_1(t, \tau) d_n - n \int_0^T R_1(t, y) d_{n-1}(y, \tau) dy. \tag{40}$$

Considering (31), (39) and (40) can be rewritten in the following form:

$$d_n(t, \tau) = d_n a_i \Phi^T(t-z) -$$

$$-n \sum_{i=1}^{m} a_i \int_{t_{i-1}}^{t_i} \Phi^T(t-y) d_{n-1}(y, \tau) dy, \quad t_{i-1} < t < t_i,$$

$$d_{n+1} = d_n \Phi^T(0) \sum_{i=1}^{m} a_i (t_i - t_{i-1}) - \tag{41}$$

$$- n \sum_{i=1}^{m} a_i \int_0^T dt \int_{t_{i-1}}^{t_i} \Phi^T(t-y) d_{n-1}(y, t) dy.$$

It can be shown that

$$d_n(t, \tau) = A(\tau) b_n(t, \tau), \tag{42}$$

where $b_n(t, \tau)$ are continuous functions for $t \neq \tau$.

By using (42), Eq. (36) can be simplified and reduced to the form

$$\sigma^T(t, t_1, \ldots, T) =$$

$$= \int_0^T L(t, \tau) B(\tau) d\tau = \sum_{i=1}^{T} b_i \int_{t_{i-1}}^{t_i} L(t, \tau) d\tau, \tag{43}$$

where

$$L(t, \tau) = \frac{R(t, \tau)}{A(\tau)} \tag{44}$$

is a regular function [9].

In practical calculations, one must always take the finite number of terms in the numerator and the denominator of (37). We can derive equations which make it possible to estimate the error resulting from this procedure.

It is known [8] that

$$|d_n| \leqslant n^{\frac{n}{2}} (MT)^n,$$

$$|d_n(t, \tau)| \leqslant \frac{1}{T} (n+1)^{\frac{n+1}{2}} (MT)^{n+1},$$

where M is the maximum value of the modulus of function (34), which is found from (31), (7), and (20) in each concrete case.

Let n_0 be an integer which satisfies the condition

$$n_0 > (MTe)^2. \tag{46}$$

Then by using (45), we can derive the error estimates:

$$\left|\sum_{n=n_0}^{\infty}\frac{d_n}{n!}\right| \leqslant \sum_{n=n_0}^{\infty}\left|\frac{d_n}{n!}\right| \leqslant \frac{(MTe)^{n_0}}{\sqrt{n_0^{n_0}}}\,\frac{1}{\sqrt{n_0}-MTe},$$

$$\sum_{n=n_0}^{\infty}\left|\frac{d_n(t,\tau)}{n!}\right| \leqslant \frac{1}{T}\,\frac{(MTe)^{n_0+1}}{\sqrt{(n_0+1)^{n_0+1}}}\,\frac{1}{\sqrt{n_0+1}-MTe}. \tag{47}$$

Let us now assume that the expression for $\sigma^T(t, t_1, \ldots, T)$ has been found. Then, in order to determine the unknown times t_i, it is necessary to use conditions which determine the transition from one portion of the linear characteristic to another ("switching conditions") [10].

For instance, consider the simplest case wherein the transition takes place when the σ coordinate attains a certain given value.

Then, the following condition must be satisfied:

$$\sigma^T(t_i) = \sigma_{i\,ass}, \tag{48}$$

where $\sigma_{i\,ass}$ are certain assigned numbers.

By using (43), we obtain

$$\sigma^T(t_1, t_1, \ldots, t_{m-1}, T) = \sigma_{i\,ass} \tag{49}$$

Expressions (49) represent the equations of periods which were to be determined. For more complex switching conditions [10], the equations of periods are composed in a similar manner.

If a computer is used for the solution of system (49), the considerations in [4] remain valid. In this, it is necessary to take into account that the periodic solutions of system (18) do not actually correspond to all systems of numbers t_i which formally satisfy Eqs. (49). It is necessary that $t_i < t_j$ for $i < j$, and that σ attains its value $\sigma_{i\,ass}$ for the first time for $t = t_i$. Moreover, for physical reasons, there may be some requirements imposed on the derivative of σ, etc. [10].

Let us briefly consider the case of a symmetric periodic regime.

Let us assume that the number of transitions is $m = 2n$, and that

$$t_k - t_{k-1} = t_{k+n} - t_{k+n-1}, \quad k < n. \tag{50}$$

The nonlinear characteristic must be such that

$$a_{n+k} = a_k, \quad b_{n+k} = -b_k, \quad k < n. \tag{51}$$

We shall seek the odd periodic regime characterized by the conditions

$$x_s^T(t) = -x_s^T\left(t + \frac{T}{2}\right), \quad \sigma^T(t) = -\sigma^T\left(t + \frac{T}{2}\right). \tag{52}$$

If (50) and (51) are satisfied,

$$A(t) = A\left(t + \frac{T}{2}\right), \quad B(t) = -B\left(t + \frac{T}{2}\right), \tag{53}$$

and, consequently, by taking (52) into account,

$$f(t) = -f\left(t + \frac{T}{2}\right).$$

By using Eq. (15) and following the above reasoning, we arrive at the integral equation

$$\sigma^T(t) = \int_0^{\frac{T}{2}} \Psi^T(t-\tau)\,A(\tau)\,\sigma^T(\tau)\,d\tau + \int_0^{\frac{T}{2}} \Psi(t-\tau)\,B(\tau)\,d\tau. \tag{54}$$

The solution of this equation is written analogously to (43):

$$\sigma^T\left(t, t_1, \ldots, t_{m-1}, \frac{T}{2}\right) = \int_0^{\frac{T}{2}} \bar{L}_1(t, \tau)\,B(\tau)\,d\tau. \tag{55}$$

Thus, in correspondence with (50), the number of undetermined times which have to be found is reduced by half. The equations of periods are composed as in the general case.

The derivation of equations in this paragraph is given in Appendix II.

4. Forced Oscillations of a Steady-State System Having a Single Nonlinearity

Consider a system of equations differing from (18) by the presence of a certain period T on the right-hand sides of the periodic time functions:

$$x_k = \sum_{\alpha=1}^{n} b_{k\alpha}x_\alpha + h_k f(\sigma) + f_k(t), \quad (k = 1, 2, \ldots, n).$$
$$\sigma = \sum j_s x_s, \quad f_k(t) = f_k(t+T), \tag{56}$$

We shall seek periodic solutions of the system (56) having the period T. The solution type is considered to be given beforehand. In contrast to the self-oscillation case, the initial instant of time corresponding to the periodic regime will be unknown here. We shall assume that the periodic regime (21) is determined. Then, by consideration of, instead of (56), the linear system (1), wherein

$$F_k(t) = h_k f(\sigma^T(t)) + f_k(t), \tag{57}$$

and by repeating the reasoning in the previous paragraph, we arrive at the integral equation

$$\sigma^T(t) = \int_0^T \Phi^T(t-\tau)\,A(\tau)\,\sigma^T(\tau)\,d\tau + \int_0^T \Phi^T(t-\tau)\,B(\tau)\,d\tau + M_1(t), \tag{58}$$

where

$$M_1(t) = \sum_{s=1}^{n}\sum_{k=1}^{n} j_s \int_0^T \Phi_{ks}^T(t-\tau)\,f_k(\tau)\,d\tau \tag{59}$$

is a known function.

By applying the superposition principle and the results of the preceding paragraph, we find

$$\sigma^T(t,\ t_0,\ldots,\ t_{m-1}) = \int_0^T L(t,\ \tau)\,B(\tau)\,d\tau + M_1(t) + \int_0^T L(t,\ \tau)\,A(\tau)\,M_1(\tau)\,d\tau. \tag{60}$$

By use of the switching conditions we find the equations of periods with respect to m unknowns $t_0, t_1, \ldots, t_{m-1}$.

It is known that the action of disturbances within the period T on a nonlinear system under certain conditions can cause the appearance of periodic motions within the period pT, where p is an integer ("subharmonic oscillations").

The described method can readily be applied to the case of subharmonic oscillations. Actually, the $f_k(t)$, $A(t)$, and $B(t)$ functions can be considered to have the pT period. Therefore, the following integral equation is valid for the σ^{pT} subharmonic oscillations:

$$\sigma^{pT}(t) = \int_0^{pT} \Phi^{pT}(t-\tau)\,A(\tau)\,\sigma^{pT}(\tau)\,d\tau + \int_0^{pT} \Phi^{pT}(t-\tau)\,B(\tau)\,d\tau + M_1(t), \tag{61}$$

where

$$\Phi^{pT}(y) = \frac{1}{pT}\sum_{m=-\infty}^{\infty} \frac{M\left(\frac{\psi_m}{p}\right)}{D\left(\frac{\psi_m}{p}\right)}\,e^{\psi_m y}. \tag{62}$$

The solution of Eq. (61) is written similarly to that of (60). By using this solution and the switching conditions, we find equations of periods for the subharmonic regime.

5. Approximate Determination of the Periodic Regime in the Form of a Trigonometric Series

Let us first consider the case of self-oscillations. In all practical calculations, we can use a finite number of terms in series (28). In this, the integral equation (33) is transformed into an equation having a degenerate kernel [8] capable of exact solution.

Actually, let

$$\tilde{\sigma}^T(t) = \int_0^T \tilde{\Phi}^T(t-\tau)\,A(\tau)\tilde{\sigma}^T(\tau)\,d\tau + \int_0^T \tilde{\Phi}^T(t-\tau)\,B(\tau)\,d\tau, \tag{63}$$

where

$$\tilde{\Phi}^T(t-\tau) = \frac{1}{T}\sum_{k=-n}^{n} \frac{M(\psi_k)}{D(\psi_k)}\,e^{\psi_k(t-\tau)}. \tag{64}$$

From (64) we can conclude that the solution of Eq. (63) is of the following form:

$$\tilde{\sigma}^T(t) = \sum_{i=-n}^{n} s_i e^{\psi_i t}, \tag{65}$$

where s_i are constants to be determined. By substituting (65) in (63) and equating the coefficients of equal harmonics, we obtain

$$s_i = \frac{M(\psi_i)}{D(\psi_i)}\sum_{k=-n}^{n} s_k\alpha_{i-k} + \frac{M(\psi_i)}{D(\psi_i)}\beta_i, \tag{66}$$

where

$$\alpha_k = \frac{i}{2\pi k}\sum_{s=1}^{m} a_s\left(e^{-\psi_k t_s} - e^{-\psi_k t_{s-1}}\right),$$

$$\beta_k = \frac{i}{2\pi k}\sum_{s=1}^{m} b_s\left(e^{-\psi_k t_s} - e^{-\psi_k t_{s-1}}\right) \tag{67}$$

are the coefficients of the Fourier function (31).

After solving the linear system (66), we find

$$s_i(t_1, t_2, \ldots, T) = \frac{1}{\tilde{\Delta}} \sum_{k=-n}^{n} \frac{M(\psi_k)}{D(\psi_k)} b_k \tilde{\Delta}_{ki}, \quad (68)$$

where $\tilde{\Delta}$ is the determinant of system (66):

$$\tilde{\Delta} = \left| \delta_{ks} - \frac{M(\psi_k)}{D(\psi_k)} \alpha_{k-s} \right|, \quad (69)$$

and $\tilde{\Delta}_{ki}$ are the corresponding cofactors.

By substituting (68) in (65), we find $\tilde{\sigma}^T(t, t_1, t_2, \ldots, T)$ which, together with the switching conditions, yields the system of periodic equations which was to be found. In particular, for the case (48), we have

$$\sum_{k=-n}^{n}{}' s_k(t_1, t_2, \ldots, T) e^{\psi_k t_i} = \sigma_{i\,\text{ass}} \quad (i = 1, 2, \ldots, m). \quad (70)$$

In the case of the odd symmetric regime, the number of time unknowns will correspondingly decrease, while equations (66) will contain only quantities s_i with odd subscripts.

In the case of forced oscillations, the approximate integral equation obtained from (58) will be of the following form:

$$\tilde{\sigma}^T(t) = \int_0^T \tilde{\Phi}^T(t-\tau) A(\tau) \tilde{\sigma}^T(\tau)\, d\tau + \int_0^T \tilde{\Phi}^T(t-\tau) B(\tau)\, d\tau + M_1(t). \quad (71)$$

Its solution will be

$$\tilde{\sigma}^T(t) = M_1(t) + \sum_{i=-n}^{n} \bar{s}_i e^{\psi_i t}, \quad (72)$$

where the values of $\bar{s}_i$ are determined by the relations

$$\bar{s}_i = -m_i + \frac{1}{\Delta} \sum_{k=-n}^{n} \left(\frac{M(\psi_k)}{D(\psi_k)} b_k + m_k \right) \tilde{\Delta}_{ki}, \quad (73)$$

and m_i are Fourier coefficients in the expansion

$$M_1(t) = \sum_{i=-\infty}^{\infty} m_i e^{\psi_i t}. \quad (74)$$

By using (73), (72), and the switching conditions, we obtain the equations of periods.

6. The General Case of a Sectionally Linear System

Let there be a system which, for $t_{i-1} < t < t_i$, is described by the equations

$$\dot{x}_k = \sum_{\alpha=1}^{n} B^i_{k\alpha} x_\alpha + f^i_k(t) \quad (k = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m), \quad (75)$$

where $B^i_{k\alpha}$ and f^i_k are certain given time functions.

We shall introduce the periodic functions

$$B_{k\alpha}(t) = B_{k\alpha}(t+T), \quad B_{k\alpha}(t) = B^i_{k\alpha}(t) \quad \text{for} \quad t_{i-1} < t < t_i,$$
$$\varphi_k(t) = \varphi_k(t+T), \quad \varphi_k(t) = f^i_k(t) \quad \text{for} \quad t_{i-1} < t < t_i. \quad (76)$$

Then, if the periodic regime (21) has been attained in system (75), we have

$$\dot{x}^T_k = \sum_{\alpha=1}^{n} B_{k\alpha}(t) x^T_\alpha + \varphi_k(t) \quad (k = 1, 2, \ldots, n). \quad (77)$$

After we separate the constant terms from the Fourier expansion of the $B_{k\alpha}(t)$ function, (77) will be written thus:

$$\dot{x}^T_k = \sum_{\alpha=1}^{n} \bar{b}_{k\alpha} x^T_\alpha + \sum_{\alpha=1}^{n} \bar{B}_{k\alpha}(t) x^T_\alpha + \varphi_k(t) \quad (k = 1, 2, \ldots, n), \quad (78)$$

where

$$\bar{b}_{k\alpha} = \frac{1}{T} \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} B^i_{k\alpha}(\tau)\, d\tau, \quad \bar{B}_{k\alpha}(t) = B_{k\alpha}(t) - \bar{b}_{k\alpha}. \quad (79)$$

Assuming that

$$F_k(t) = \sum_{\alpha=1}^{n} \bar{B}_{k\alpha} x^T_\alpha(t) + \varphi_k(t) \quad (k = 1, 2, \ldots, n) \quad (80)$$

and by using (5), we arrive at the Fredholm system of integral equations of the second kind:

$$x^T_s(t) = \sum_{k=1}^{} \int_0 R_{ks}(t, \tau) x_k(\tau)\, d\tau + P_s(t), \quad (81)$$

where

$$R_{ks}(t,\ \tau) = \sum_{m=1}^{n} \Phi_{mk}^{T}(t-\tau)\,\overline{B}_{ms}(\tau), \qquad (82)$$

$$P_s(t) = \sum_{s=1}^{n} \int_0^T \Phi_{ks}^{T}(t-\tau)\,\varphi_k(\tau)\,d\tau. \qquad (83)$$

Here, in contrast to cases previously considered, the $\Phi_{\alpha s}^{T}$ functions depend on the unknown times t_i which, however, does not involve any difficulties in principle.

As is known [8 and 9], the system of equations (81) can be directly reduced to a single equation with arguments t and τ, which are defined within the square given by $0 \leq t \leq nT$ and $0 \leq \tau \leq nT$. Therefore, the method for solving system (77) does not differ from those described above. In particular, as in Section 5, system (77) can always be reduced to the degenerate equation.

After finding the solution

$$x_s^T = x_s^T(t,\ t_0, \ldots,\ T)$$

by use of the switching conditions, which can be most diverse in dependence on the design and designation of the controlled system, we arrive at the equations of periods. In particular, if we consider the system

$$\dot{x}_k = \sum_{\alpha=1}^{n} b_{k\alpha}x_\alpha + h_k a_i(t)\,\sigma + b_i(t) \quad \text{for} \quad t_{i-1} < t < t_i, \qquad (84)$$

instead of (56), we find that all the results obtained in Sections 3, 4, and 5 can be almost entirely applied to this system with the exception of the fact that instead of function (31), the following functions must be considered:

$$\overline{A}(t) = \overline{A}(t+T), \quad \overline{A}(t) = a_i(t) \quad \text{for} \quad t_{i-1} < t < t_i,$$
$$B(t) = \overline{B}(t+T), \quad \overline{B}(t) = b_i(t) \quad \text{for} \quad t_{i-1} < t < t_i. \qquad (85)$$

APPENDIX I

Derivation of Equations in Section 2

Let the Fourier expansion of the $F_k(t)$ function be of the following form:

$$F_k(t) = \sum_{r=-\infty}^{\infty} f_{kr}\,e^{\psi_r t}. \qquad (86)$$

We shall seek the periodic solution of system (1) in the form of a Fourier series:

$$x_s^T(t) = \sum_{k=-\infty}^{\infty} \chi_{sk}\,e^{\psi_k t}. \qquad (87)$$

By substituting (86) and (87) in (1) and by equating the coefficients of equal harmonics, we obtain

$$\chi_{sm} = -\sum_{k=1}^{n} \frac{f_{km}\,\Delta_{ks}(\psi_m)}{D(\psi_m)}. \qquad (88)$$

Considering that

$$f_{km} = \frac{1}{T}\int_0^T F_k(\tau)\,e^{-\psi_m \tau}\,d\tau, \qquad (89)$$

by substitution of (89) and (88) in (87) and by changing the order of summation and integration, we obtain (5).

Let us further consider the Fourier expansion of the function of the period T, which coincides with the $\Phi_{ks}(y)$ function for $0 < y < T$. We have

$$\frac{1}{T}\int_0^T \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \frac{e^{\lambda_\rho \tau}}{e^{\lambda_\rho T} - 1}\,e^{-\psi_p \tau}\,d\tau = \qquad (90)$$
$$= \frac{1}{T}\sum_{\rho=1}^{n} \frac{\Lambda_{ks}(\lambda_\rho)}{D'(\lambda_\rho)} \frac{1}{\lambda_\rho - \psi_p} = -\frac{1}{T}\frac{\Delta_{ks}(\psi_p)}{D(\psi_p)},$$

which proves Eq. (7).

Equation (12) can be proved in a similar manner if we take into account that

$$\frac{1}{(\psi_k - \lambda_\rho)^\nu} = \frac{(-1)^\nu}{(\nu-1)!}\int_0^T \frac{d^{\nu-1}}{d\lambda_\rho^{\nu-1}}\left(\frac{e^{\lambda_\rho \tau}}{e^{\lambda_\rho T} - 1}\right) e^{-\psi_k \tau}\,d\tau. \qquad (91)$$

From (7), we further have

$$\Phi_{ks}(T) - \Phi_{ks}(0) = \sum_{\rho=1}^{n} \frac{\Delta_{ks}(\lambda_\rho)}{D'(\lambda_\rho)}, \qquad (92)$$

whence, by taking into account the well-known equations from [11], we obtain (9).

Equation (8) can be obtained by using the following well-known property of Fourier series:

$$\Phi_{ks}^{T}(0) = \Phi_{ks}^{T}(T) = \frac{\Phi_{ks}(T) + \Phi_{ks}(0)}{2}. \qquad (93)$$

Equations (15)–(17) are proved in a similar manner on the basis of the fact that, in the case (14), the Fourier expansions of the $F_k(t)$ functions will not contain even harmonics.

APPENDIX II

Proof of Equations in Section 3

The recurrent kernels for Eq. (33) will be calculated by using the equation

$$R_n(t,\ \tau)=\int_0^T R_1(t,\ y)\,R_{n-1}(y,\ \tau)\,dy. \qquad (94)$$

By induction, it can be shown that

$$R_n(t,\ \tau)=A(\tau)\,L_n(t,\ \tau), \qquad (95)$$

where

$$L_1=\Phi^T(t-\tau),\ \ldots,$$

$$L_n=\sum_{i=1}^{n}a_i\int_{t_{i-1}}^{t_i}\Phi^T(t-y)\,L_{k-1}(y,\ \tau)\,dy \qquad (96)$$

are continuous functions for $n > 1$.

Considering that $d_n(t, \tau)$ is a linear combination of $R_1, \ldots, R_n$, we obtain (42), whence, by taking into account the absolute convergence of the numerator in (37), we obtain (44). Further, by multiplying Eq.(33) by A(t), we find that the function

$$s(t)=c^T(t)\,A(t) \qquad (97)$$

satisfies the equation

$$s(t)=\int_0^T A(t)\,\Phi^T(t-\tau)\,s(\tau)\,d\tau+\int_0^T A(t)\,\Phi^T(t-\tau)\,B(\tau)\,d\tau, \qquad (98)$$

whose free term is a function which can be represented by the kernel

$$N_1(t,\ \tau)=A(t)\,\Phi^T(t-\tau). \qquad (99)$$

The solution of Eq. (98) can be represented in the following form:

$$s(t)=\int_0^T N(t,\ \tau)\,B(\tau)\,d\tau, \qquad (100)$$

where $N(t,\tau)$ is the resolvent of kernel (99).

By direct calculation, it can be shown that the recurrent kernels N_k are equal to

$$N_k(t,\ \tau)=A(t)\,L_k(t,\ \tau) \qquad (101)$$

and therefore

$$N(t,\ \tau)=A(t)\,L(t,\ \tau). \qquad (102)$$

By substituting (102) and (97) in (100), we obtain (43). It now remains to prove the estimates (47). From the first of the equations (45), we have

$$s_{n_0}=\left|\sum_{k=n_0}^{\infty}d_k\right|\leqslant\sum_{k=n_0}^{\infty}\frac{k^{\frac{k}{2}}M}{k!}. \qquad (103)$$

Since, according to Stirling's formula [12],

$$n!>\left(\frac{n}{e}\right)^n, \qquad (104)$$

then,

$$s_{n_0}<\sum_{k=n_0}^{\infty}\left(\frac{MTe}{\sqrt{k}}\right)^k<\sum_{k=n_0}^{\infty}\left(\frac{MTe}{\sqrt{n_0}}\right)^k. \qquad (105)$$

If we assume that n_0 satisfies condition (46), then, by summing the geometric progression on the right-hand side of (105), we obtain the first of the equations (47). The residue of the numerator in (37) is estimated in a similar manner.

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ON THE THEORY OF TWO-CHANNEL FOLLOW-UP SYSTEMS WITH A RELAY COMPONENT IN THE AC CIRCUIT

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It is shown that the describing-function method, when applied to two-channel systems with modulation and a nonlinear element in the ac circuit, becomes an exact method for determining self-oscillations for an unbounded increase in the carrier frequency. An approximate criterion for the self-oscillation stability in such systems with a relay component is formulated. The transfer function of a system which is linearized by self-oscillations with respect to slow control inputs is determined.

In ac follow-up systems, the infralow-frequency error signal is transformed into a signal with the carrier frequency Ω, after which it is amplified and then again transformed into an infralow-frequency signal. Alternating-current follow-up systems where the infralow-frequency portion of the circuit has two channels, in particular, two identical channels and antisymmetrical cross couplings [1], are used to a considerable extent. Some automatic ac bridges [2], some follow-up hydraulic platforms, etc., belong to these systems. If the ac factors and the two-channel infralow-frequency part of the circuit are linear, such systems can be conveniently investigated by the method of complex transfer functions [1] with the application of ordinary frequency criteria for stability and performance. However, in a number of cases, two-channel relay systems with a relay element in the ac signal circuit can be recommended. A similar situation occurs in the case of considerable and unavoidable changing of the ac channel amplification factor and in a number of other cases. In such systems, the basic operating conditions are the conditions of "spatial" self-oscillations of rather high frequency and small amplitude. With respect to control inputs, which change slowly, the system behaves as a linear system due to vibrational linearization [5 and 6] by self-oscillations.

The purpose of the present article is the presentation of a relatively general method for the investigation of similar two-channel systems with a relay element in the ac circuit, as well as the analysis of certain common properties of these systems.

The method under consideration is suitable for the solution of actual technical problems.

1. Structural Diagram and the System Transfer Functions

The quantities $x_{1\,in}$ and $x_{2\,in}$ are supplied to the input of a two-channel follow-up system (Fig. 1). The output quantities $x_{1\,out}$ and $x_{2\,out}$ are subtracted from these quantities, and the following error signals are formed:

$$\varepsilon_1 = x_{1\,in} - x_{1\,out} \qquad \varepsilon_2 = x_{2\,in} - x_{2\,out}$$

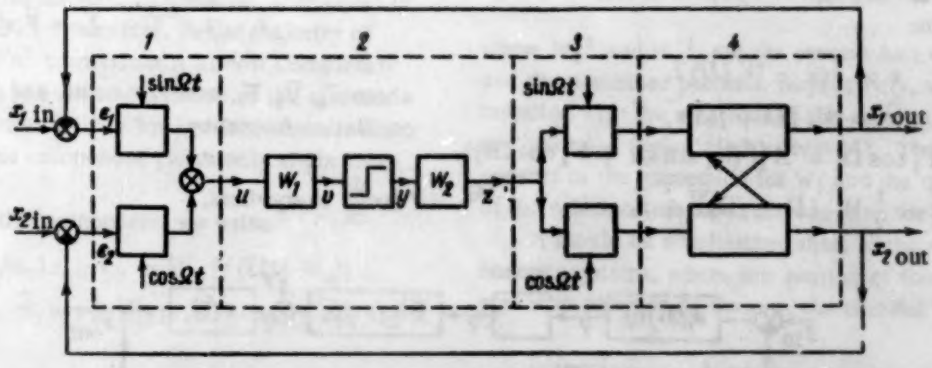

Fig. 1. General diagram of a two-channel follow-up system with a relay element in the ac circuit.

The error signals are fed to modulator 1, where they are transformed into a signal with the carrier frequency Ω:

$$u = \varepsilon_1 \sin \Omega t + \varepsilon_2 \cos \Omega t = \varepsilon \sin (\Omega t + \varphi).$$

This signal is transmitted through the first linear portion of the ac circuit, which has the transfer function* $W_1(D)$, and then through the relay element and the second linear portion, whose transfer function is $W_2(D)$.

The output signal $\underline{z}$ of the single-channel portion of the ac circuit is fed to the demodulator 3, where it is multiplied by sin Ωt and cos Ωt. The z sin Ωt and z cos Ωt quantities enter the two-channel infralow-frequency part of circuit 4.

We introduce the complex quantities

$$\bar{x}_{in} = x_{1\,in} + x_{2\,in} \qquad \bar{x}_{out} = x_{1\,out} + jx_{2\,out}$$
$$\bar{z} = z \sin \Omega t + jz \cos \Omega t = jze^{-j\Omega t}, \qquad (1)$$

and we write

$$\bar{\varepsilon} = \bar{x}_{in} - \bar{x}_{out} \quad \bar{x}_{out} = W(D)\bar{z}. \qquad (2)$$

Taking into account the possible presence of antisymmetrical cross couplings in the two-channel portion of the circuit, we shall consider W(D) as a complex function in the general case.

The equations of the instantaneous values of variables in the circuit single-channel portion are of the following form:

$$u = \varepsilon \sin (\Omega t + \varphi), \quad v = W_1(D)u,$$
$$y = f(v), \qquad z = W_2(D)y, \qquad (3)$$

where f(v) is the nonlinear factor characteristic.

The single-channel portion of the circuit consists of an ac circuit, and the variables in this circuit can be conventionally represented by complex amplitudes.

We shall assume that the variable $x = X_1 \sin \Omega t + X_2 \cos \Omega t$ has the complex "amplitude" $\bar{X}_1 = X_1 + jX_2$, which is a function of time in the general case.

In the Appendix it is shown that the complete system of equations in terms of complex variables is of the following form:

$$\bar{\varepsilon} = \bar{x}_{in} - W(D)\bar{z},$$
$$\bar{V} = W_1(D + j\Omega)\bar{\varepsilon},$$
$$Y_1 \sin \Omega t + Y_2 \cos \Omega t = A[f(V_1 \sin \Omega t + V_2 \cos \Omega t)],$$
$$\bar{z} = \frac{1}{2} W_2(D + j\Omega)\bar{Y} + \bar{z}'_g. \qquad (4)$$

Here, the A[f] symbol denotes the first harmonic of the almost periodic function $f(V_1 \sin \Omega t + V_2 \cos \Omega t)$. Thus, $\bar{Y} = Y_1 + jY_2$ represents the complex "amplitude" of the carrier frequency first harmonic at the nonlinear element output, to the input of which a carrier frequency signal with the complex "amplitude" $\bar{V} = V_1 + jV_2$ is supplied.

The frequency spectrum of the z'_g component begins in the vicinity of the carrier frequency Ω and then extends toward higher frequencies.

Equations (4) correspond to the structural diagram with complex transfer functions and complex variables given in Fig. 2. The following fact should be noted here. The transfer functions of the single-channel linear portions of the circuit for the envelope in the considered system with a two-channel infralow-frequency portion are obtained by the simple substitution of $D + j\Omega$ for the instantaneous values of the D operator in the transfer functions, where Ω is the carrier frequency (the factor $\frac{1}{2}$ in front of W_2 is generated by the demodulator–synchronous detector).

Such a simple rule for the formation of the transfer function for the envelope is valid only for systems with identical channels in the infralow-frequency two-channel portion of the circuit. Only in such systems can the sinusoidal as well as the cosinusoidal output signal components in the ac channel be used to the same extent. As is known [3], in ordinary ac follow-up systems with a single-channel infralow-frequency portion, the formation of the transfer function for the envelope is somewhat more complicated.

2. Steady-State Self-Oscillations

As was mentioned above, self-oscillations represent the basic operating conditions of relay systems of the class under consideration.

In order to determine steady-state self-oscillations, we shall assume that $\bar{x}_{in} = 0$, and we shall seek the solution of system (4) in the following form:

$$\bar{\varepsilon} = \bar{\varepsilon}_0 e^{j\omega_a t}, \quad \bar{V} = \bar{V}_0 e^{j\omega_a t},$$
$$\bar{z} = \bar{z}_0 e^{j\omega_a t}, \quad \bar{Y} = \bar{Y}_0 e^{j\omega_a t}, \qquad (5)$$

where $\bar{\varepsilon}_0$, $\bar{V}_0$, $\bar{z}_0$, and $\bar{Y}_0$ = const, and ω_a is the self-oscillation frequency.

* $D = \frac{d}{dt}$ everywhere.

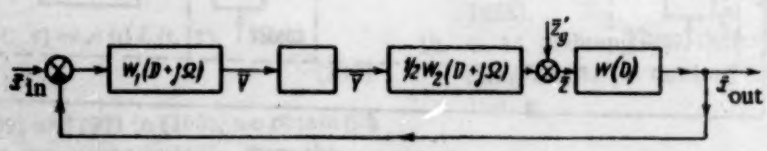

Fig. 2.

Due to the presence of the $\bar{z}'_g$ term, this is an approximate solution, and its form entirely corresponds to the describing-function method [4].

However, in application to the considered systems with a relay element in the ac circuit, the describing-function method has a special feature. This feature, which renders the describing-function method an exact method when applied to such systems, consists in the following. Here, instead of neglecting harmonics with frequencies which are multiples of the self-oscillation frequency, we neglect harmonics with frequencies which are multiples of the carrier frequency Ω or of the combined frequency $\Omega \pm \omega_a$.

Actually, for $\bar{x}_{in} = 0$, by substituting expressions (5) in equations (4), we find

$$\begin{gathered}\bar{\varepsilon}_0 = -W(j\omega_a)\bar{z}_0, \\ \bar{V}_0 = W_1(j\omega_a + j\Omega)\bar{\varepsilon}_0, \\ \bar{Y}_0 = W\bar{V}_0, \\ \bar{z}_0 = \frac{1}{2}W_2(j\omega_a + j\Omega)\bar{Y}_0 + \bar{z}'_g e^{-j\omega_a t}\end{gathered} \tag{6}$$

Here, $W_n = \frac{\bar{Y}_0}{\bar{V}_0}$ is the amplitude response of the nonlinear element, which is determined by the relation

$$\begin{gathered}Y_1 \sin \Omega t + Y_2 \cos \Omega t = Y_0 \cos(\omega_a t + \varphi_1)\sin \Omega t + \\ + Y_0 \sin(\omega_a t + \varphi_1)\cos \Omega t = Y_0 \sin(\omega_a t + \Omega t + \varphi_1) = \\ = A[f(V_1 \sin \Omega t + V_2 \cos \Omega t)] = A\{f[V_0 \sin(\omega_a t + \Omega t + \varphi)]\}, \\ \bar{Y}_0 = Y_0 e^{j\varphi_1}, \quad \bar{V}_0 = V_0 e^{j\varphi}.\end{gathered}$$

For a relay element without an insensitivity zone and hysteresis, which will mainly be considered below,

$$W = \frac{4a}{\pi}\frac{1}{V_0}, \tag{7}$$

where

$$a = \text{const}.$$

The $\bar{z}'_g e^{-j\omega_a t}$ component, the neglecting of which renders the describing-function method an approximate method, contains higher harmonics. From Eqs. (26) and (27) in the Appendix, it follows that, in the given case, the lowest frequency of these harmonics is equal either to $\Omega + \omega_a$ (the nonlinear element response is nonsymmetrical, and there is a constant component at the ac channel output) or $2(\Omega + \omega)$, while, in ordinary cases where the describing-function method is used, the lowest frequency is $2\omega_a$.

In the majority of practical cases, the carrier frequency Ω is considerably higher than the possible self-oscillation frequency. Thus, for angle-tracking systems with conical scanning, Ω is equal to $(60\pi - 200\pi)$ sec^{-1}. For ac systems with standard frequencies, Ω is equal to $(100\pi - 800\pi)$ sec^{-1}. Therefore, in the majority of cases, the $\bar{z}'_g e^{-j\omega_a t}$ component is almost completely suppressed in the two-channel portion of the circuit, which always acts as a filter for low frequencies, and the neglect of this component yields only slight inaccuracies.

Neglecting this component, we write

$$\begin{gathered}\bar{\varepsilon}_0 = -W(j\omega_a)\bar{z}_0, \quad \bar{V}_0 = W_1[j(\Omega + \omega_a)]\bar{\varepsilon}_0, \\ \bar{Y}_0 = W_n\bar{Y}_0, \quad \bar{z}_0 = \frac{1}{2}W_2[j(\Omega + \omega_a)]\bar{Y}_0.\end{gathered} \tag{8}$$

Hence,

$$\bar{V}_0 = -\frac{1}{2}W(j\omega_a)W_1[j(\Omega + \omega_a)]W_2[j(\Omega + \omega_a)]W_n\bar{V}_0$$

or

$$1 + W_1(j\omega_a)W_n = 0. \tag{9}$$

Here, the quantity

$$W_1(j\omega_a) = \frac{1}{2}W(j\omega_a)W_1[j(\Omega + \omega_a)]W_2[j(\Omega + \omega_a)]$$

can be called the over-all transfer locus of the system linear portion, and the quantity

$$W_1(D) = \frac{1}{2}W(D)W_1(D + j\Omega)W_2(D + j\Omega)$$

can be called the complex transfer function of the linear portion.

Equation (9) and the equations equivalent to it,

$$W = -W_n^{-1}, \quad W_1^{-1} = -W_n,$$

where W_1^{-1} and W_n^{-1} are the inverse loci of the linear and the nonlinear portions, respectively, whose shapes coincide with the relations of the describing function method in L. S. Gol'dfarb's form [4]. The difference consists in the expression for W_1 and the transformation of the relation into an exact equation for $\Omega \to \infty$.

It should be emphasized that, in the considered two-channel systems, where one portion of the circuit operates with alternating current, the transfer locus,

$$W_1(j\omega) = \frac{1}{2}W(j\omega)W_1(j\omega + j\Omega) \times W_2(j\omega + j\Omega)$$

is generally not symmetric with respect to the real axis (i.e., $W_1(j\omega) \neq W_1(-j\omega)$) even if there are no antisymmetrical cross couplings between identical elements in the circuit two-channel portion. Therefore, the transfer locus must be considered in the range where ω_a changes from $-\infty$ to ∞. As usual, the values of ω_a at the points of intersection between $W_1(j\omega_a)$ and W_n^{-1} (or between $W_1^{-1}(j\omega_a)$ and W_n) correspond to the frequencies of possible self-oscillations, and the $W_n = W_n(V_0)$ values at these points determine the amplitudes V_0 of possible self-oscillations (Fig. 3).

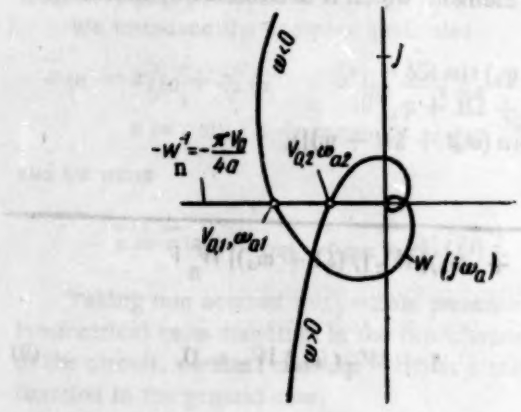

Fig. 3.

3. Stability of Self-Oscillations

In order to find the conditions for the stability of self-oscillations we shall represent them in the form

$$\bar{\varepsilon} = (\bar{\varepsilon}_0 + \overline{\Delta\varepsilon})\, e^{j\omega_a t},\ \bar{V} = (\bar{V}_0 + \overline{\Delta V})\, e^{j\omega_a t},$$
$$\bar{Y} = (\bar{Y}_0 + \overline{\Delta Y})\, e^{j\omega_a t},\quad \bar{z} = (\bar{z}_0 + \overline{\Delta z})\, z^{j\omega_a t}, \tag{10}$$

where, as above, the quantities with the "0" subscripts correspond to the amplitudes of steady-state oscillations, and the second terms on the right-hand side represent deviations. Thereafter, we shall consider the deviations to be small, i.e., we shall consider stability in Lyapunov's sense.

In order to write the equations in terms of deviations, it is necessary to establish a relation between $\overline{\Delta Y}$ and $\Delta\bar{V}$ at the relay element output and input.

For the relay element under consideration,

$$Y_1 = \frac{4}{\pi}\, a\, \frac{V_1}{V},\quad Y_2 = \frac{4}{\pi}\, a\, \frac{V_2}{V},$$

where

$$V = \sqrt{V_1^2 + V_2^2}.$$

Hence, with an accuracy to small higher orders, we obtain

$$\Delta Y_1 = \frac{4}{\pi}\, a \left[\frac{\Delta V_1}{V_0} - \frac{V_1}{V_0^3}\,(V_1\Delta V_1 + V_2\Delta V_2)\right],$$
$$\Delta Y_2 = \frac{4}{\pi}\, a \left[\frac{\Delta V_2}{V_0} - \frac{V_2}{V_0^3}\,(V_1\Delta V_1 + V_2\Delta V_2)\right].$$

However,

$$V_1 = V_0 \cos(\omega_a t + \varphi),\ V_2 = V_0 \sin(\omega_a t + \varphi).$$

Therefore,

$$\overline{\Delta Y} = \Delta Y_1 + j\Delta Y_2 =$$
$$= \frac{4a}{\pi}\left\{\frac{\overline{\Delta V}}{V_0} - \frac{e^{j(\omega_a t+\varphi)}}{V_0}\right\}\cos(\omega_a t + \varphi)\,\Delta V_1$$
$$+ \sin(\omega_a t + \varphi)\,\Delta V_2\Big\} = \frac{4a}{\pi V_0}\left[\overline{\Delta V} - \right.$$
$$\left.\frac{e^{2j(\omega_a t+\varphi)}+1}{2}\,\Delta V_1 + j\,\frac{e^{2j(\omega_a t+\varphi)}-1}{2}\,\Delta V_2\right] =$$
$$= \frac{2a}{\pi V_0}\left[\overline{\Delta V} - \overline{\Delta V}^{*} e^{2j(\omega_a t+\varphi)}\right]. \tag{11}$$

Here, $\overline{\Delta V}^{*}$ is the complex conjugate of $\overline{\Delta V}$.

By substituting expressions (10) in (4) and considering (11), for $\bar{x}_{in} = 0$, we find

$$\overline{\Delta\varepsilon} = -W(D + j\omega_a)\,\overline{\Delta z},$$
$$\overline{\Delta V} = W_1(D + j\omega_a + j\Omega)\,\overline{\Delta\varepsilon},$$
$$\overline{\Delta z} = \frac{1}{2}\,W_2(D + j\omega_a + j\Omega)\,\overline{\Delta Y} + \bar{z}_g' e^{-j\omega_a t},\tag{12}$$
$$\overline{\Delta Y} = \frac{2a}{\pi V_0}\left[\Delta\bar{V} - \Delta\bar{V}^{*} e^{2j(\omega_a t+\varphi)}\right].$$

Thus, in the general case, the equations for the self-oscillation deviations contain a periodically changing coefficient $e^{2j(\omega_a t+\varphi)}$. If the self-oscillations vary slowly, and this is the most typical case, $\overline{\Delta V}$ and $\overline{\Delta V}^{*}$ are functions which vary slowly in comparison with $e^{2j(\omega_a t+\varphi)}$, and the effect of the second term on the right-hand side of the expression for $\overline{\Delta Y}$ can be neglected. The $\bar{z}_g' e^{j\omega_a t}$ term, which, according to what has been said above, contains higher frequencies, can be even more readily neglected.

In connection with this, the approximate equation of deviations of self-oscillations, which is obtained from Eq. (12), is of the following form:

$$\left[1 + \frac{2a}{\pi V_0}\,W_1(D + j\omega_a)\right]\overline{\Delta V} = 0, \tag{13}$$

where

$$W_1(D + j\omega_a) = \frac{1}{2}\,W(D + j\omega_a)$$
$$W_1(D + j\omega_a + j\Omega)\,W_2(D + j\omega_a + j\Omega)$$

is the over-all transfer function of the circuit linear portion for the envelope of the self-oscillation frequency ω_a.

If we know the equation of deviations, we can use any criterion applicable to equations with complex coefficients for determining the stability conditions. By applying the Nyquist criterion, we find: if the transfer function $W_1(D)$ of the linear portion of an open-loop system does not have poles in the right half-plane, those and only those self-oscillations are stable for which the points $(\pi/2a)V_0$, j0 (V_0 is the self-oscillation amplitude) are not encompassed by the transfer locus $W_1(j\omega)$ when ω changes from $-\infty$ to ∞.

By comparing this condition with Eq. (9) for steady-state self-oscillations and the corresponding graphic plot, we readily arrive at a different formulation of the self-oscillation stability criterion for the relay system under consideration: if (-1, j0) is the intersection point between W_1 and W_n^{-1} that corresponds to the given self-oscillations, then, for a $W_1(D)$ which does not have poles in the right half-plane and if ω varies from $-\infty$ to ∞, these self-oscillations will be stable only if the transfer locus $W_1(j\omega)$ does not encompass the $(-21, j0)$ point, whose distance from the imaginary axis is twice as large in comparison with the first point. Thus, the self-oscillations with the ω_{a1} and ω_{a2} frequencies in Fig. 4 are unstable, and self-oscillations with the ω_{a3} and ω_{a4} frequencies are stable. Hence, it follows that self-oscillations with the largest amplitude are always stable.

It must be emphasized that the above stability criterion for self-oscillations corresponds to the approximate equation (13) and is valid insofar as this equation is valid.

Another fact should be noted. As can be seen from the above, stable self-oscillations with a positive as well as negative frequency ω_a can exist in the systems under consideration. The sign of ω_a in two-channel systems corresponds to the sense of rotation of the tracing point or to the direction of the actual three-dimensional conical motion in two-channel follow-up systems with mechanical output quantities. The frequency ω_a can have the same sign as the carrier frequency Ω, or it can be of opposite sign. In angle-tracking systems with conical scanning, the first case corresponds to the adding of the scanning and self-oscillation angular velocities, and the second case corresponds to the subtraction of these velocities. In any system of the class under consideration, the first case corresponds to the operation of the circuit single-channel portion at the combined frequency, and the second case corresponds to operation at the frequency difference.

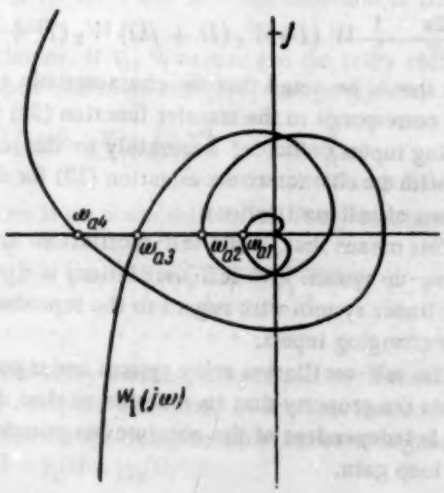

Fig. 4.

The realization of one or the other direction of self-oscillations depends on the initial phase shift in the circuit (the phase shift for $\omega_a = 0$) and the shape of $W_1(j\omega)$.

The change in the initial phase shift, produced, for instance, by changing the carrier frequency phase in the modulator, means that the entire transfer locus $W_1(j\omega)$ is rotated in the corresponding direction. In such rotation, the self-oscillations in one of the two possible directions can always be rendered unstable (see Fig. 4).

4. Reproduction of Control Inputs

With regard to the reproduction of control inputs, which change slowly in comparison with self-oscillations, the behavior of the two-channel systems under consideration is analogous to other vibrationally linearized nonlinear systems [5, 6]. For small deviations, the dynamic properties of such self-oscillation systems are close to the characteristics of linear systems.

In order to find the transfer function of a self-oscillation system with respect to the slowly changing control input $\bar{x}_{in}$, we shall write the variables in the following form:

$$\bar{\varepsilon} = \bar{\varepsilon}_0 e^{j\omega_a t} + \overline{\Delta\varepsilon},\ \bar{V} = \bar{V}_0 e^{j\omega_a t} + \overline{\Delta V},$$
$$\bar{Y} = \bar{Y}_0 e^{j\omega_a t} + \overline{\Delta Y},\ \ \bar{z} = \bar{z}_0 e^{j\omega_a t} + \overline{\Delta z},$$

where the first term on the right-hand side in each expression corresponds to self-oscillations, and the second term, according to conditions, changes slowly in comparison with the first.

By substituting these expressions in equations (4), we find

$$\bar{\varepsilon}_0 e^{j\omega_a t} + \overline{\Delta\varepsilon} = \bar{x}_{in} - W(j\omega_a)\bar{z}_0 e^{j\omega_a t} - W(D)\overline{\Delta z},$$
$$\bar{V}_0 e^{j\omega_a t} + \overline{\Delta V} = W_1(j\Omega + j\omega_a)\bar{\varepsilon}_0 e^{j\omega_a t} + W_1(D + j\Omega)\overline{\Delta\varepsilon}, \quad (14)$$
$$\bar{z}_0 e^{j\omega_a t} + \overline{\Delta z} = \frac{1}{2}W_2(j\Omega + j\omega_a)\bar{Y}_0 e^{j\omega_a t} + W_2(D + j\Omega)\overline{\Delta Y} + \ddot{z}_g;\ Y_0 \sin(\Omega t + \omega_a t + \varphi) + \Delta Y_1 \sin\Omega t + \Delta Y_2 \cos\Omega t = A\{f[V_0 \sin(\Omega t + \omega_a t + \varphi) + \Delta V_1 \sin\Omega t + \Delta V_2 \cos\Omega t]\}. \quad (15)$$

In the considered case of a relay element with the ideal characteristic $f(x) = a\,\mathrm{sign}(x)$, the first harmonics

in the expansion of the f function are expressed by the well-known equations [7]

$$f[V_0 \sin(\Omega t + \omega_a t + \varphi) + \Delta V_1 \sin \Omega t + \Delta V_2 \cos \Omega t] = a \operatorname{sign}[V_0 \sin(\Omega t + \omega t + \varphi) + \Delta V \sin(\Omega t + \varphi_1)] = \frac{4a}{\pi} N \sin(\Omega t + \omega_a t + \varphi) + \frac{2a}{\pi} \frac{\Delta V}{V_0} M \sin(\Omega t + \varphi_1) + \ldots \ldots \ldots \ldots, \quad (16)$$

where

$$\varphi_1 = \operatorname{arctg} \frac{\Delta V_2}{\Delta V_1},$$

$$N = \frac{2}{\pi} E\left(\frac{\Delta V}{V_0}\right) = 1 - \frac{1}{4}\left(\frac{\Delta V}{V_0}\right)^2 + \ldots,$$

$$M = \frac{4}{\pi}\left[\frac{V_0^2}{\Delta V^2} E\left(\frac{\Delta V}{V_0}\right) - \left(\frac{V_0^2}{\Delta V^2} - 1\right) K\left(\frac{\Delta V}{V_0}\right)\right] = 1 + \frac{1}{8}\left(\frac{\Delta V}{V_0}\right)^2 + \ldots,$$

and E and K are complete elliptic integrals of the first and the second kind, respectively.

Here, the quantities with the "0" subscript–the self-oscillation amplitudes–are always constant, and the increments ΔV, $\Delta \epsilon$, etc. change slowly or are constant.

From (15) and (16), it follows:

$$\bar{Y}_0 = \frac{4a}{\pi} N \frac{\bar{V}_0}{V_0}$$

or

$$\bar{Y}_0 = W_n' \bar{V}_0, \quad (17)$$

where

$$W_n' = \frac{4a}{\pi} \frac{1}{V_0} N\left(\frac{\Delta V}{V_0}\right).$$

It also follows from (15) and (16) that

$$\Delta \bar{Y} = \frac{2a}{\pi} \frac{M}{V_0} \Delta \bar{V}. \quad (18)$$

By equating the coefficients in front of $e^{j\omega_a t}$ and the slowly changing terms in equations (14) and by eliminating the intermediate variables by means of expressions (17) and (18), we find.

$$1 + W_1(j\omega_a) W_n' = 0, \quad (19)$$

$$\bar{x}_{out} = \frac{\frac{2a}{\pi V_0} W_1'(D)}{1 + \frac{2a}{\pi V_0} W_1'(D)} \bar{x}_{in} \quad (20)$$

Here,

$$W_n' = \frac{2a}{\pi} \frac{N}{V_0} = W_n N,$$

$$W_1 = \frac{1}{2} W(D) W_2(D + j\Omega) M W_1(D + j\Omega),$$

and $W_1(j\omega_a)$ has the previous value:

$$W_1(j\omega_a) = \frac{1}{2} W(j\omega_a) W_1(j\Omega + j\omega_a) W_2(j\Omega + j\omega_a).$$

In the general case, for large $\frac{\Delta V}{V_0}$ deviations, the N and M quantities basically depend on these deviations.

In this, Eqs. (19) and (20) constitute a system of interdependent nonlinear equations, and the self-oscillation amplitude depends on the control input.

However, the basic system operating conditions are those conditions for which the deviations, which are caused by control inputs, are considerably smaller than the self-oscillation amplitude, i.e., $\Delta V/V_0 < 1$. Under these conditions, it can be considered that N = 1 and M = 1.

Actually, even if $\frac{\Delta V}{V_0} = 0.5$, N = 0.95 and M = 1.02. Thus, in the $0 \leq \frac{\Delta V}{V_0} \leq 0.5$ interval, the N = 1 and M = 1 equalities are satisfied very accurately. If these equalities are satisfied, Eq. (19) is transformed into the equation for steady-state self-oscillations,

$$1 + W_1(j\omega_a) W_{n\Phi} = 0, \quad (21)$$

which has been considered above, and Eq. (20) corresponds to the transfer function of the linear follow-up system,

$$\Phi(D) = \frac{\frac{2a}{\pi V_0} W_1(D)}{1 + \frac{2a}{\pi V_0} W_1(D)}, \quad \bar{x}_{out} = \Phi(D) \bar{x}_{in} \quad (22)$$

which, in the open-loop state, has the transfer function,

$$\frac{2a}{\pi V_0} W_1(D) = \frac{2a}{\pi V_0} \frac{1}{2} W(D) W_1(D + j\Omega) W_2(D + j\Omega).$$

It should be noted that the characteristic equation which corresponds to the transfer function (22) for slowly changing inputs coincides accurately to the $j\omega_a$ term for D with the characteristic equation (13) for the establishment of self-oscillations.

This means that, if the self-oscillations are stable, a follow-up system with self-oscillations is similar to a stable linear system with regard to the reproduction of slowly changing inputs.

The self-oscillation relay system under consideration has the property that its reaction to slow disturbances is independent of the absolute magnitude of the open-loop gain.

Actually, it follows from Eq. (9) that, if the modulus of the open-loop gain changes slowly, the self-oscillation amplitude V_0 increases in proportion to the gain:

$$W_{\text{п}}^{-1} = \frac{\pi}{4a} V_0 = -W_1(j\omega_a).$$

The $\frac{2a}{\pi V_0} W_1(D)$ quantity, which enters Eq. (22), remains constant, which determines the invariability of the closed-loop system transfer function.

Here, it should be added that, if only the gain of the first factor W_1 of the circuit single-channel portion changes, which occurs often in practice, then not only is the over-all transfer function constant, but also the amplitude ϵ_0 of self-oscillations at the point of error measurement is constant (V_0 changes).

In conclusion, it should be mentioned that this method and the considered facts can be relatively simply applied to the case of a relay element with hysteresis and an insensitivity zone.

APPENDIX

It is known [8], that the transfer function for complex amplitudes (for the carrier frequency Ω envelopes) is obtained from the transfer function for instantaneous values by substituting D+jΩ for the D operator.

Thus,

$$\overline{V} = W_1(D + j\Omega)\,\overline{U}, \quad \overline{U} = \overline{\varepsilon}. \tag{23}$$

The quantity,

$$y = f(v) = f(V_1 \sin \Omega t + V_2 \cos \Omega t),$$

where $\overline{V} = V_1 + jV_2$, can always be represented in the form

$$y = f(V_1 \sin \Omega t + V_2 \cos \Omega t) = Y^0 + Y_1 \sin \Omega t + Y_2 \cos \Omega t + \sum_{i=2}^{\infty} (Y_1^{(i)} \sin i\Omega t + Y_2^{(i)} \cos i\Omega t),$$

where Y^0, Y_1, Y_2, $Y_1^{(i)}$, and $Y_2^{(i)}$ are functions of time in the general case.

In particular, if V_1, V_2 = const and the relay element has a z-shaped characteristic (f(v) = a sign v), then

$$Y^0 = 0, \quad Y = \sqrt{Y_1^2 + Y_2^2} = \frac{4}{\pi} a,$$

$\frac{Y_1}{V_1} = \frac{Y_2}{V_2}$ (there is no phase shift in the relay element), and

$$Y_1^{(2)} = Y_2^{(2)} = 0, \quad Y^{(3)} = \sqrt{(Y_1^{(3)})^2 + (Y_2^{(3)})^2} = \frac{4}{3\pi} a, \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{24}$$

Thus, the relay element output can be characterized by the complex amplitude of the first harmonic $\overline{Y} = Y_1 + jY_2$ and by the complex amplitudes of higher harmonics $\overline{Y}^{(i)} = Y_1^{(i)} + jY_2^{(i)}$.

In correspondence with this, the complex amplitude of the first harmonic of the factor W_2 output will be expressed by the equation,

$$\overline{Z} = W_2(D + j\Omega)\,\overline{Y}, \tag{25}$$

and the complex amplitudes of higher harmonics of this quantity will be determined by the relations,

$$\overline{Z}^{(i)} = W_2(D + ij\Omega)\,\overline{Y}^{(i)}.$$

The instantaneous value Z of the ac channel output is equal to

$$z = Z_1 \sin \Omega t + Z_2 \cos \Omega t + z_g, \tag{26}$$

where $Z_1 + jZ_2 = \overline{Z}$, and z_g is the component corresponding to high harmonics and the component of the zero carrier frequency (if $Y^0 \neq 0$, also such a component is present).

However, according to (1) and (26),

$$\bar{z} = ze^{-j\Omega t} = (Z_1 \sin \Omega t + Z_2 \cos \Omega t + z_g) e^{-j\Omega t} = \frac{1}{2}(Z_1 + jZ_2) - \frac{1}{2}(Z_1 - jZ_2) e^{-2j\Omega t} + jz_g e^{-j\Omega t}$$

or

$$\bar{z} = \frac{1}{2}\bar{Z} + \bar{z}_g', \tag{27}$$

Here,

$$\bar{z}_g' = jz_g e^{-j\Omega t} - \frac{1}{2}(Z_1 - jZ_2) e^{-2j\Omega t}$$

is the component which, under steady-state conditions, has components with a frequency of the order of Ω and higher. This follows from the fact that z_g contains harmonics with double and higher frequencies. Thus, the closed systems of equations for the circuit under consideration is of the following form:

$$\bar{\varepsilon} = \bar{x}_{\text{in}} - W(D)\,\bar{z},$$
$$\overline{V} = W_1(D + j\Omega)\,\bar{\varepsilon}, \tag{28}$$
$$Y_1 \sin \Omega t + Y_2 \cos \Omega t = f(V_1 \sin \Omega t + V_2 \cos \Omega t) + \sum,$$
$$\bar{z} = \frac{1}{2} W_2(D + j\Omega)\,\overline{Y} + \bar{z}_g',$$

where Σ denotes higher harmonics.

Example. Consider the case where

$$W_1(D) = \text{const}, \quad W_2(D) = e^{-\tau D}, \quad W = \frac{1}{D(1 + TD)}.$$

This case corresponds to the inertialess first element in the ac channel (for instance, a broadband amplifier), to an element with a constant lag at the relay-ele-

ment output (in particular, it can reflect the actuation time of an actual relay), and to a two-channel final element in the form of inertial and integration elements which are connected in series (for instance, electric motors).

Considering that a phase shift of the reference signal can occur in the modulator (or in the demodulator), we assume that

$$W_1(D) = \bar{k}_1,$$

where $\bar{k}_1$ is generally a complex quantity.

Thus, in the example under consideration, the overall transfer function of the linear portion is:

$$W_1(D) \frac{1}{2} W(D) W_1(D + j\Omega) W_2(D + j\Omega) =$$
$$= \frac{1}{2} \frac{\bar{k}_1 e^{-\tau(D+j\Omega)}}{D(1+TD)} = \frac{\bar{k} e^{-\tau D}}{D(1+TD)},$$

where

$$\bar{k} = \frac{1}{2} \bar{k}_1 e^{-j\tau\Omega}.$$

For a relay element, expression (7) holds, and Eq.(9) for the self-oscillation frequencies and amplitudes assumes the following form:

$$\frac{\bar{k} e^{-j\tau\omega_a}}{j\omega_a(1+jT\omega_a)} = -\frac{\pi V_0}{a}.$$

The corresponding graphic plot is shown in Fig. 5. The root locus portions corresponding to $\tau\omega_a > \pi$ are not shown, since the approximation of dynamic properties by means of a constant-lag element ceases to be valid in the region of frequencies higher than π/τ for a relay and many other elements.

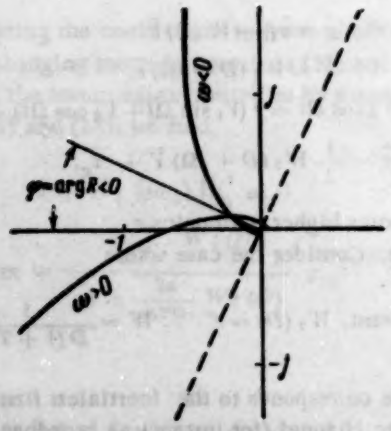

Fig. 5. Root locus of $W_1(j\omega)$ for the example under consideration.

In dependence on the $\varphi = \arg \bar{k}$ values, it is obvious that one or two self-oscillation regimes can exist in the system under consideration, which differ from each other by the magnitude and sign of ω_a (the direction of conical motion) and their amplitude.

The self-oscillation equations in terms of real quantities are of the following form:

$$\tau\omega_a + \operatorname{arc\,tg} T\omega_a = \pm\frac{\pi}{2} + \varphi,$$

$$V_0 = \frac{4ak}{\pi|\omega_a|\sqrt{1+T^2\omega_a^2}}.$$

If $\tau = 0$, self-oscillations with finite frequency can occur only for $\varphi \neq 0$, i.e., if some cross coupling is present. Under these conditions, for $\pi/2 > \varphi > 0$, only self-oscillations with negative frequency can exist:

$$\omega_a = \frac{1}{T} \operatorname{arc\,tg}\left(\varphi - \frac{\pi}{2}\right),$$

and for $-(\pi/2) < \varphi < 0$, only self-oscillations with positive frequency can occur:

$$\omega_a = \frac{1}{T} \operatorname{arc\,tg}\left(\varphi + \frac{\pi}{2}\right).$$

As follows from the above, the self-oscillations are stable in both cases. For $T = 0$ (only integrating elements are present in the system two-channel portion):

$$\omega_a = \frac{\pm\pi/2 + \varphi}{\tau}, \quad V_0 = \frac{4ak}{\pi|\omega_a|}.$$

In this case, self-oscillations with positive as well as negative frequency can exist either for $\varphi = 0$ or $\varphi \neq 0$.

However, it follows from the above self-oscillation stability criterion that, for $\varphi > \pi/6$, only self-oscillations with negative frequency are stable, and, for $\varphi < -(\pi/6)$, only self-oscillations with positive frequency are stable.

In reproducing slowly changing control inputs, the self-oscillation system under consideration has, in accordance with (22), the following transfer function:

$$\Phi(D) = \frac{\frac{2a\bar{k}}{\pi V_0} e^{-\tau D}}{D(1+TD) + \frac{2a\bar{k}}{\pi V_0} e^{-\tau D}} \bar{x}$$

In particular, for $T = 0$, because of the relation $V_0 = \frac{4ak}{\pi|\omega|}$, the transfer function assumes the following form:

$$\Phi(D) = \frac{\frac{1}{2}|\omega_a| e^{-\tau D + j\varphi}}{D + \frac{1}{2}|\omega_a| e^{-\tau D + j\varphi}} = \frac{|\omega_a| e^{j\varphi}}{2De^{\tau D} + |\omega_a| e^{j\varphi}}$$

Here, ω_a depends only on the phase shift φ and the lag τ:

$$\omega_a = \frac{\pm\pi/2 + \varphi}{\tau}.$$

Thus, the transfer function for slowly changing control inputs is invariant with respect to changes in the coefficient $\bar{k}$ modulus.

This illustrates the general aspect of the determined self-adjustment property of the considered self-oscillation systems.

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ON THE DYNAMIC CHARACTERISTICS OF HYDRAULIC COUPLINGS

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A method for the calculation and experimental determination of the dynamic frequency-response curves for hydraulic couplings is presented. A brief description of the equipment used in experiments and the results of the experimental determination of characteristics of some couplings are provided.

Hydraulic couplings in the shape of pipes filled with a liquid are used in automatic control systems (ACS). In designing ACS, the dynamic properties of such couplings must be taken into account. A method for the calculation and experimental determination of the dynamic frequency-response curves for hydraulic couplings is given below.

I. A. Charnyi [1] performed a theoretical investigation of the effect of piping on the accuracy of the pressure gauge which registers pressure variations. The author based his investigation on the N. E. Zhukovskii equations of the hydraulic shock theory [2], which had an additional term whereby the pressure-head loss due to friction was taken into account.

Following I. A. Charnyi [1], our theoretical investigation of the dynamic frequency response of hydraulic couplings will also be based on these equations:

$$-\frac{\partial p}{\partial x}=\rho\left(\frac{\partial v}{\partial t}+2av\right), \qquad -\frac{\partial p}{\partial t}=K\frac{\partial v}{\partial x}, \tag{1}$$

$$K=\frac{K_0}{1+\frac{K_0 d}{E\delta_0}}, \qquad a=\frac{16\nu}{d^2}, \tag{2}$$

where p and v are the pressure and velocity at the pipe section which is at the distance x from the origin, respectively, ρ is the liquid density, K_0 is the actual elasticity modulus of the liquid, E is Young's modulus of the first kind for the pipe material, δ_0 is the pipe wall thickness, d is the pipe inside diameter, and ν is the liquid kinematic viscosity coefficient.

The dynamic characteristics will be determined for two cases: for piping without discharge and for piping with liquid discharge.

For the first case, the boundary conditions will be written in the following form:

$$x=0,\ p=p_0e^{i\omega t}, \tag{3}$$

$$x=l,\ v=0. \tag{4}$$

For the second case, considering that the discharge velocity is approximately proportional to pressure, i.e., v = Mp, we obtain the following boundary conditions:

$$x=0,\quad p=p_0e^{i\omega t}, \tag{5}$$

$$x=l,\quad v=Mp, \tag{6}$$

where l is the piping length, and M is the proportionality factor.

We shall consider only forced oscillations, and we shall seek partial solutions of Eq. (1) of the following form:

$$p=(A\cos nx+B\sin nx)\,e^{i\omega t}, \tag{7}$$

$$v=(C\cos nx+D\sin nx)\,e^{i\omega t}. \tag{8}$$

The values of A, B, C, D, and n are found from Eqs. (1), (7), and (8) and boundary conditions (3)-(6).

Let us consider the first case. Here, $A=p_0$, $B=-p_0\operatorname{tg}nl$, $C=\frac{i\omega}{nK}p_0\operatorname{tg}nl$, $D=-\frac{i\omega}{nK}p_0$,

$$n=\frac{1}{c}(\omega'-ia'),$$

where $$\omega'=\sqrt{\frac{\sqrt{\omega^4+4a^2\omega^2}+\omega^2}{2}},$$

$$a'=\sqrt{\frac{\sqrt{\omega^4+4a^2\omega^2}-\omega^2}{2}},$$

$c=\sqrt{\frac{K}{\rho}}$ is the sound velocity in the piping.

If a is small in comparison with ω, $a'=0$, and $\omega'=\omega$.

By substituting the values of A, B, and n in (7), for x = l, we obtain:

$$p_l=(p_0\cos nl+p_0\operatorname{tg}nl\sin nl)\,e^{i\omega t}. \tag{9}$$

After transforming (9), we find

$$p_l = \frac{p_0 e^{i\omega t}}{\cos\frac{\omega' l}{c}\operatorname{ch}\frac{a' l}{c} + i\sin\frac{\omega' l}{c}\operatorname{sh}\frac{a' l}{c}}$$

or

$$p_l = p_0 A(\omega)\, e^{i[\omega t - \varphi(\omega)]}, \tag{10}$$

where

$$A(\omega) = \frac{1}{\sqrt{\operatorname{ch}^2\frac{a' l}{c} - \sin^2\frac{\omega' l}{c}}} \tag{11}$$

is the amplitude response, and

$$\varphi(\omega) = \operatorname{arc\,tg}\left(th\frac{a' l}{c}\operatorname{tg}\frac{\omega' l}{c}\right) \tag{12}$$

is the phase response.

It follows from (11) and (12) that the amplitude response $A(\omega)$ has not only one, but a number of resonance maxima for

$$\frac{\omega' l}{c} = \frac{\pi}{2} + m\pi, \tag{13}$$

where m = 0, 1, 2, . . .

Consequently, the resonance frequencies will be

$$\omega'_p = (0.5 + m)\frac{c}{2l}. \tag{14}$$

With an increase in oscillation frequency, $\operatorname{ch}\frac{a' l}{c}$ in (11) will also increase, due to which the amplitude response will have its greatest value in the first resonance maximum. The amplitude response in the subsequent resonance maxima will gradually decrease.

Figure 1 shows the amplitude response $A(\omega)$ and the phase response $\varphi(\omega)$ curves, which were calculated according to Eqs. (11) and (12) for the following case:

$l = 1000$ cm, $\nu = 1.0$ cm²/sec, $\delta_0 = 0.1$ cm, $\rho = 0.8\frac{g\,sec^2}{cm^4}$, $d = 0{,}4$ cm, $E = 0.8\cdot 10^6$ kg/cm², $K_0 = 14\,000$ kg/cm².

It is obvious from Fig. 1 that, for frequencies of up to 200 cps, the amplitude response has three resonance maxima: at frequencies of 10, 73, and 134 cps. At the resonance maxima, the phase-shift angle is equal to $\frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi$, respectively.

For the case with liquid discharge, by using boundary conditions (5) and (6), we obtain: $A = p_0$, $n = \frac{1}{c}(\omega' - ia')$ and

$$B = p_0 \frac{\frac{i\omega}{nK}\sin nx + M\cos nx}{\frac{i\omega}{nK}\cos nx - M\sin nx}.$$

By substituting the values of A, B, and n in (7), we obtain after transformations:

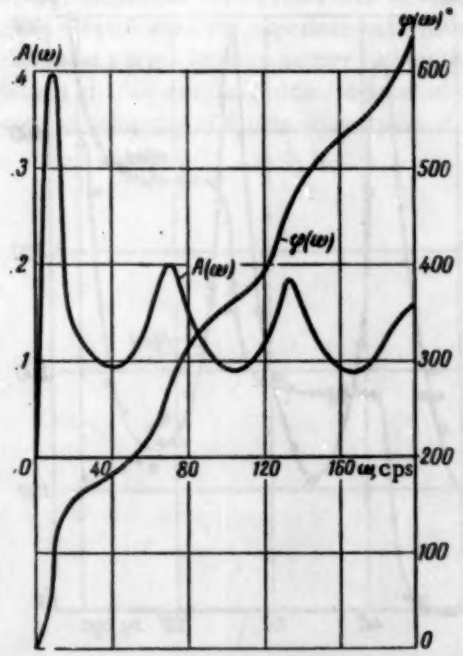

Fig. 1.

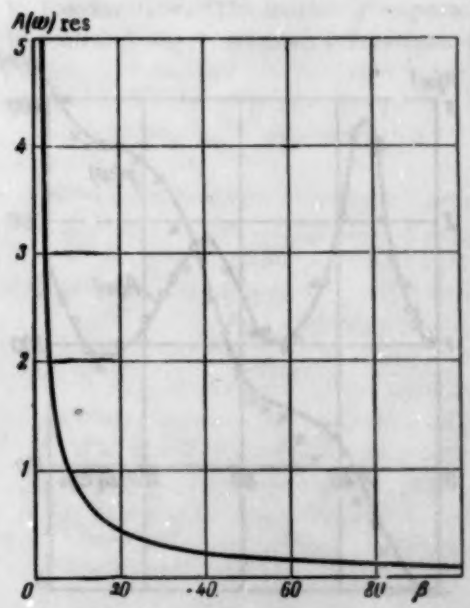

Fig. 2.

$$p_l = \frac{p_0 e^{i\omega t}}{\cos\frac{\omega' l}{c}\operatorname{ch}\frac{a' l}{c} - \beta(\omega_1 P - a_1 Q) + i\left[\sin\frac{\omega' l}{c}\operatorname{sh}\frac{a' l}{c} + \beta(\omega_1 Q - a_1 P)\right]}, \quad (15)$$

where $\beta = \frac{MK}{c}$,

$$a_1 = \sqrt{\frac{\sqrt{\omega^2 + 4a^2} + \omega}{2\omega}}, \quad \omega_1 = \sqrt{\frac{\sqrt{\omega^2 + 4a^2} - \omega}{2\omega}},$$

$$P = \sin\frac{\omega' l}{c}\operatorname{ch}\frac{a' l}{c}, \quad Q = \cos\frac{\omega' l}{c}\sin\frac{a' l}{c}.$$

If a is small in comparison with ω, $a_1=1$ and $\omega_1=0$. Thus, in the given case,

$$p_l = p_0 \frac{e^{i\omega t}}{r + iS} = p_0 A(\omega) e^{i[\omega t - \varphi(\omega)]}, \quad (16)$$

where the values of r and S are clear from expression (15).

The amplitude response is determined by the equation

$$A(\omega) = \frac{1}{\sqrt{r^2 + S^2}}, \quad (17)$$

and the phase response is given by the equation

$$\varphi(\omega) = \operatorname{arc\,tg}\frac{S}{r}. \quad (18)$$

For low-viscosity liquids (for instance, kerosene), the value of a is small. In this case, the approximate equation for $A(\omega)$ is of the following form:

$$A(\omega) = \frac{1}{\sqrt{1 - \sin^2\frac{\omega l}{c}(1 - \beta^2)}}. \quad (19)$$

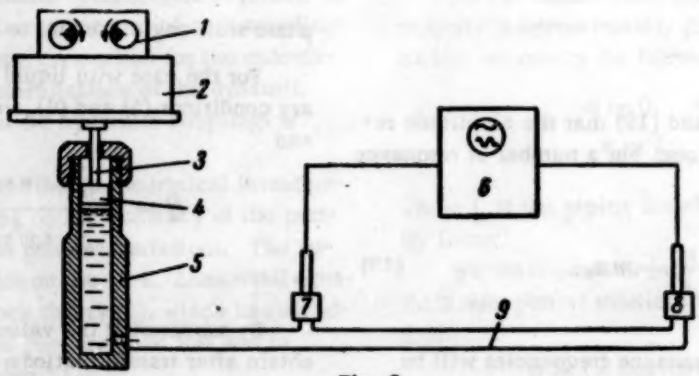

Fig. 3.

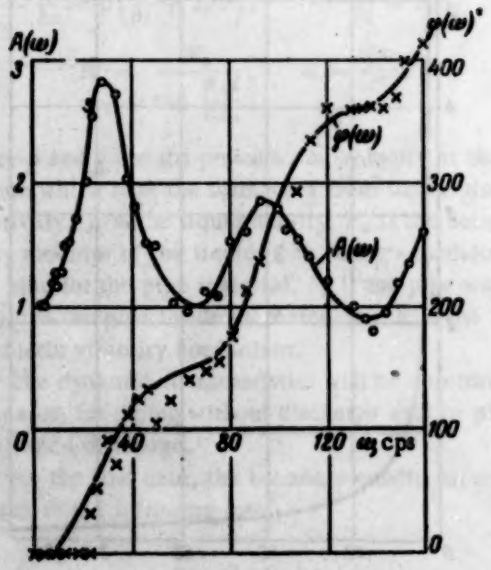

Fig. 4.

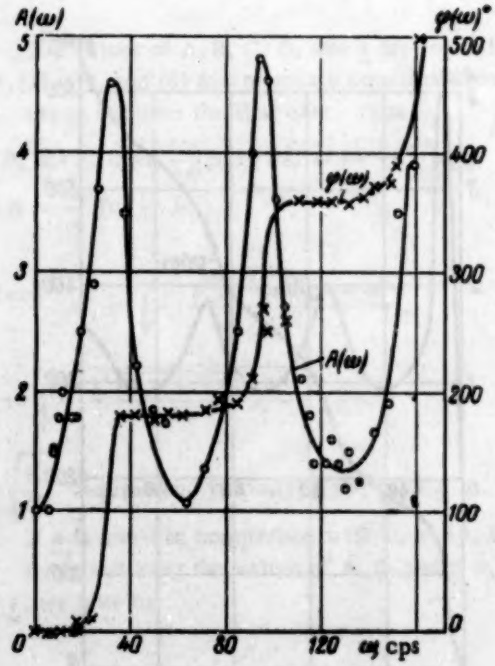

Fig. 5.

It follows from (19) that, with an increase in β, the values of $A(\omega)$ decrease. Figure 2 shows the $A(\omega)=f(\beta)$ dependence for the first resonance maximum $\frac{\omega l}{c} = \frac{\pi}{2}$, whence it can be seen that the sharpest decrease in the amplitude response occurs for $\beta = 0\text{-}8$. For $\beta > 8$, no resonance phenomena are observed with an increase in the oscillation amplitude.

The dynamic characteristics of hydraulic lines were determined by experiments where the frequency method was used [3]. The pressure oscillations at the piping inlet were secured by means of a special stand, the diagram of which is shown in Fig. 3. The stand consists of the following components: a cylinder, 5, which is filled with the liquid, a piston, 4, a rod, 3, a weight, 2, and a centrifugal vibrator, 1, which is driven by an electric motor. The rod, the weight, and the vibrator are rigidly connected to each other. Their combined weight creates a certain initial pressure in the cylinder. As the vibrator eccentric masses rotate, a harmonic disturbing force is created. By acting on weight 2, this force causes it to vibrate, due to which harmonic pressure oscillations arise in the cylinder and, consequently, in the hydraulic line 9. Centrifugal LV-500 vibrators, which were made in the Bauman MVTU, were used for this stand. A 2780-S "Orion" (Hungarian People's Rep.) two-beam pressure indicator, 6, with a piezoelectric quartz crystal was used for recording pressure oscillations at the hydraulic-line inlet and outlet. The data transmitters 7 and 8 of the pressure indicator have a high natural oscillation frequency, which attains 20 kc. Our experiments were performed in the frequency region from 0 to 160 cps. The average pressure was equal to 25-30 atm, and the total (double) amplitude varied from 6 to 20 atm.

Figures 4 and 5 show the experimental dynamic frequency-response curves for loop copper hydraulic lines with a length of 1000 cm, an inside diameter of 4 mm, and an outside diameter of 6 mm, which were filled with transformer oil (Fig. 4) and kerosene (Fig. 5). The data in Figs. 4 and 5 indicate the presence of two resonance maxima of the amplitude response. The phase-shift angle at these maxima is $\pi/2$ and $3\pi/2$. At the resonance maxima, the amplitude of pressure oscillations increases 2.8- to 4.8-fold.

For hydraulic lines filled with kerosene or transformer oil, in correspondence with Eq. (11), the frequency of the first resonance maximum for $\sin \omega l/c \approx 1$ is approximately equal to

$$\omega_p = \frac{30\,000}{l} \quad (l \text{ in cm}).$$

The frequencies of the second and the third resonance maximum will be three and five times as large, respectively. Therefore, the theoretical amplitude responses of the hydraulic lines under consideration must have resonance maxima for the 30, 90, and 150 cps frequencies, which is in good agreement with the experimental results.

Thus, the dynamic frequency-response curves for hydraulic couplings were determined theoretically and experimentally, and their flow characteristics were explained.

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AN INVESTIGATION OF SOME TYPES OF MATRIX DECODERS WITH A RELAY OUTPUT

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One possible approach to the analysis of matrix decoders made from semiconductor diodes and having a relay output is given.

Decoders which use unicombinational matrices (C_n^m) and which provide a defensive rejection (for any distortion of the code) are studied.

Relations for a selection network which interrelates the parameters of the output relays, matrix, and power supply voltage are derived for both continuous and pulse powering.

The industrial development of semiconductors, in particular, diodes with a large back resistance, permits one to build and use diode matrix circuits with relay output in telecontrol and telesignaling (TC-TS) apparatus.

The different types of such decoders are distinguished from one another by the object selection code (uni- or all-combinational matrices are used), by the means of generating the input (a voltage input or an input through relay contacts is used), by the switching method, and by the operating mode of the output relays.

The decoders mentioned, in contrast to the widely used matrix decoders with various kinds of trigger-circuit outputs, require rather large currents of prolonged duration, since their input resistances are relatively small.

Moreover, a significant number of parallel circuits through the forward and back resistances of diodes is formed in unicombinational matrix decoders, depending on the selected code, and this raises the requirements on the quality of the diodes, and increases the power consumed.

The matrix parameters, and sometimes the input voltages, will be different, depending on the required operating conditions and the relay parameters chosen.

Therefore, it is best to be able to evaluate this or that type of decoder beforehand, and to compute its parameters.

One possible approach to the investigation of matrix decoders with relay outputs is given in this article, and relations are derived for decoders having defense in their circuits (defensive rejection for any sort of code distortion), and which use unicombinational matrices (C_n^m), for two cases—continuous and pulse (through a capacitor) powering of the output relays.

The effect of a change in the decoder parameters and of certain external factors on the operation is also examined.

Relations for the Circuits of Unicombinational Matrices (C_n^m)

Let us examine a unicombinational matrix separately, before deriving the relations for the decoder.

A C_5^2 matrix is shown in Fig. 1; the generalized parameters for a unicombinational C_n^m matrix are designated on it. It follows from an examination of this circuit that the total number of input circuits is equal to the total number of combinational elements n; the number of output circuits is equal to the number of possible combinations of the assumed $N = C_n^m$; the number of input buses connected to one output bus is equal to the number of elements in each code m. The number of connections of one input bus to output buses s can be found from the formula $s = C_{n-1}^{m-1}$. The number of output circuits which are not connected to the given input circuit is equal to N-s.

An Investigation of Decoders with Continuous Powering of the Output Relays

Let us turn to the study of the decoder operation.

According to the equivalent diagram (Fig. 2, c) for the circuit under consideration, we can write

$$u = i(R_1 + R_{eq}), \tag{1}$$

where R_1 is the input resistance of the given decoder circuit, R_{eq} is the equivalent resistance of the rest of its circuits, and i is the current of the equivalent circuit.

The different operating modes of the decoder are caused by the change in the equivalent resistance R_{eq}, and current i; the resistance R_1, and applied voltage u, remain constant.

Let us explain how the change in the above-mentioned quantities is determined.

Having used the intermediate equivalent circuit (Fig. 2, b), we get

$$R_{eq} = \frac{R\,R_b R_p}{R_a R_b + R_a R_p + R_b R_p} = \frac{R_p}{1 + \frac{R_p}{R_a} + \frac{R_p}{R_b}}. \quad (2)$$

Resistances R_a and R_b will be equal to

$$R_a = (R_{fd} + R_2')\frac{1}{m}, \quad (3)$$

$$R_b = (R_{fd} + R_c)\frac{1}{n-m}. \quad (4)$$

according to the equivalent circuit of Fig. 2, a, under the condition that all elements with the same designation are identical.

Resistance R_3 in the circuit is equal to

$$R_3 = \frac{R_{fd} + R_{2(k)}}{k}.$$

The resistances R_2' and R_B, in turn, are determined from the following formulas, expressed in matrix parameters:

$$R_2' = \frac{sR_{2(m)}\left[R_{rd} + \frac{R_p(R_{fd} + R_{2(k)})}{kR_p + R_{fd} + R_{2(k)}}\right]}{R_{rd} + \frac{R_p(R_{fd} + R_{2(k)})}{kR_p + R_{fd} + R_{2(k)}} + s(s-1)R_{2(m)}} = \frac{1}{\frac{1}{sR_{2(m)}} + \frac{s-1}{R_{rd} + \frac{R_p[R_{fd} + R_{2(k)}]}{kR_p + R_{fd} + R_{2(k)}}}}, \quad (5)$$

$$R_c = \frac{1}{N-s-1}\left[R_{rd} + \frac{R_p(R_{fd} + R_{2(k)})}{kR_p + R_{fd} + R_{2(k)}}\right]. \quad (6)$$

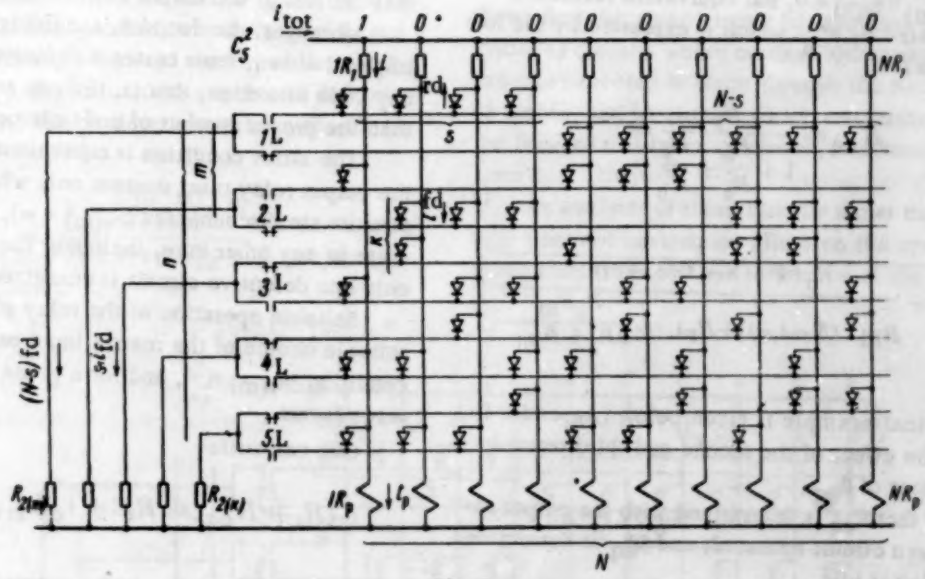

Fig. 1.

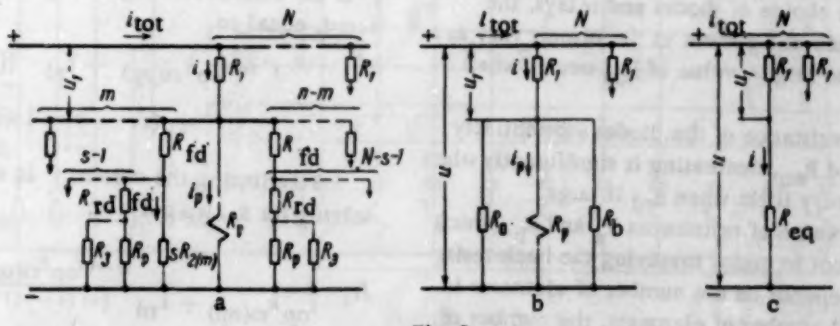

Fig. 2.

where R_{fd} and R_{rd} are the forward and back resistances of the diodes; R_p is the resistance of the relay coil; $R_{z(m)}$ is the value of the matrix resistance R_z for the input circuits pertaining to the given code; $R_{z(k)}$ is the value of resistance R_z for the defensive circuits of the matrix; k is the number of defensive circuits.

Let us find how a change in the resistance R_z effects the equivalent resistance R_{eq}, all other quantities remaining constant. It follows from formulas (2)–(5) that the equivalent resistance will equal

$$R'_{eq} = \frac{R_p}{1 + \frac{R_p}{R'_a} + \frac{R_p}{R'_b}}, \qquad (7)$$

when $R_{z(m)} = \infty$, where

$$\frac{R_p}{R'_a} = \frac{R_p m (s-1)}{R_{fd}\ (s-1) + R_{rd} + \frac{R_p (R_{fd} + R_{z(k)})}{kR_p + R_{fd} + R_{z(k)}}};$$

$$\frac{R_p}{R'_b} = \frac{R_p (N-s-1)(n-m)}{R_{fd}\ (N-s-1) + R_{rd} + \frac{R_p (R_{fd} + R_{z(k)})}{kR_p + R_{fd} + R_{z(k)}}}.$$

When $R_{z(m)} = R_{z(k)} = 0$, the equivalent resistance will be designated by R''_{eq}, which is expressed by the following formula:

$$R''_{eq} = \frac{R_p}{1 + \frac{R_p}{R'_a} + \frac{R_p}{R'_b}} = \qquad (8)$$

$$= \frac{R_p}{1 + \frac{R_p m}{R_{fd}} + \frac{R_p (N-s-1)(n-m)}{R_{fd}\ (N-s-1) + R_{rd} + \frac{R_p R_{fd}}{kR_p + R_{fd}}}}$$

A numerical example is given below (Appendix I) to evaluate the effect of the second and third terms in the denominator of R_{eq}.

We note that R'_{eq} is determined with the output relay of the given circuit operated, and R''_{eq} is determined when that relay is idle.

By examining the expressions introduced above, we can draw the following conclusions.

1) For a given choice of diodes and relays, the equivalent resistance and current in the circuit vary as a function of R_z. The largest value of R_{eq} occurs when $R_z = \infty$.

2) The back resistance of the diodes substantially effects the value of R_{eq}, decreasing it significantly when R_{rd} is small, and very little when R_{rd} is large.

The absolute value of resistances R_a and R_b, which are determined from formulas involving the back resistances of diodes, depends on the number of elements in the code, the total number of elements, the number of output circuits, and the number of connections between the buses, that is, on the chosen C_n^m.

3) If it is assumed that the back resistance is sufficiently large ($R_{rd} \approx \infty$) and the matrix resistance $R_{z(m)} = \infty$, the equivalent resistance will be equal to the relay resistance, $R'_{eq} = R_p$. The resistance, $R_{z(m)} = \infty$, is characteristic for decoder inputs coming from relay contacts; in all other cases, including voltage input (if $R_{z(m)}$ is not infinite), R_{eq} will be less than the value mentioned.

4) The equivalent resistance R''_{eq} will equal

$$R''_{eq} = \frac{R_p}{1 + \frac{R_p m}{R_{fd}}},$$

for the same diode back resistance ($R_{rd} = \infty$), and for $R_{z(m)} = R_{z(k)} = 0 \left(R_a = R_{fd}\ \frac{1}{m}\right)$, that is, it will be significantly less than R'_{eq}.

We will derive the design formulas relating the decoder parameters to the applied voltage.

When the given voltage is applied, the decoder must ensure reliable operation of the output relay of the circuit which corresponds to the code, and must ensure that the rest of the output circuits do not operate.

Moreover, the decoder, according to the condition imposed above, must ensure a defensive rejection for any code distortion, that is, if there are more or less than the proper number of code elements m.

The latter condition is equivalent to stating that the output relay must operate only when all defensive circuits are disconnected ($R_{z(m)} = \infty$), and must not operate in any other case, including the case in which only one defensive circuit is energized.

Reliable operation of the relay will be ensured by a suitable choice of the matrix input resistance, R_1, for the condition, $R_{z(m)} = \infty$, and for a given relay pull-in reserve factor.

One can write

$$i' (R_1 + R'_{eq}) = R_1 i' + i_{op} k_{rs(op)} R_p = u, \qquad (9)$$

for this case, where i_{op} is the operating (pull-in) current of the relay; $k_{rs(op)}$ is the pull-in reserve factor; i' is the current flowing through R_1 in the mode indicated, equal to

$$i' = i_{op} k_{rs(op)} + i_{rd}\ [(s-1)m + (N-s-1)(n-m)].$$

Substituting the current i' in equation (9), and solving for R_1, we get

$$R_1 \frac{u - i_{op} k_{rs(op)} R_p}{i_{op} k_{rs(op)} + i_{rd}\ [m(s-1) + (n-m)(N-s-1)]}.$$

If the back resistance of the diodes is large, the second term in the denominator can be ignored, and the approximate formula

$$R_1 \approx \frac{u}{i_{op} k_{rs(op)}} - R_p.$$

can be used.

This formula, expressed in terms of the drop-out current and reset coefficient of the relay, will take the form

$$R_1 = \frac{u k_{rt}}{i_{do} k_{rs(op)}},$$

where

$$k_{rt} = \frac{i_{do}}{i_{op}}.$$

The reverse current which flows through the diodes can be found from the formula,

$$i_{rd} = \frac{u - i_{op} k_{rs(op)} (R_p + R_1)}{R_1[m(s-1)+(n-m)(N-s-1)]}.$$

By using the relations for the second operating state of the decoder (relay unoperated, where $R_{2(m)} = 0$, and $R_{rd} = \infty$, R_{eq}), we find the forward current of the diodes. This current, $i_{fd(k)}$, depends on the number of defensive circuits formed, and can be determined from the expression

$$i_{fd(k)} = \frac{u - \frac{i_{op} k_{rt}}{k_{rs(op)}} R_1}{kR_1 + sR_2 + R_{fd}},$$

where $k = 1, 2, \ldots, m, \ldots, n$ is the number of defensive circuits; the remaining designations are the same as before.

The largest current through the diodes will occur when there is one defensive circuit. It is useful to know this current in order to choose diodes correctly, so that they will not be overloaded during prolonged operation.

The power required by the matrix decoder depends on its operating mode, and a large amount of power will be required if no code is present.

This power can be determined from the formula

$$P = i_{tot} u = \frac{u^2 N}{R_1 + R_{eq}},$$

on the basis of the equivalent circuit (Fig. 2, c), under the condition that all the circuits are identical; R_{eq} is the equivalent resistance in the absence of code, and i_{tot} is the total current required by the decoder.

An Investigation of a Decoder with Pulse Powering of the Output Relays

A matrix decoder, whose output relays are connected through capacitors, is shown in Fig. 3.

An examination of the equivalent circuit (Fig. 4, a) shows that the given circuit has parallel circuits of two types: a) circuits which contain inductances and capacitances connected to them through the back resistances of diodes, and b) circuits which are connected through the forward resistance of diodes, and contain the resistance R_2.

An analysis of these circuits shows that the first type does not have much effect on the operation of the given circuit if the back resistance of the diodes is suf-

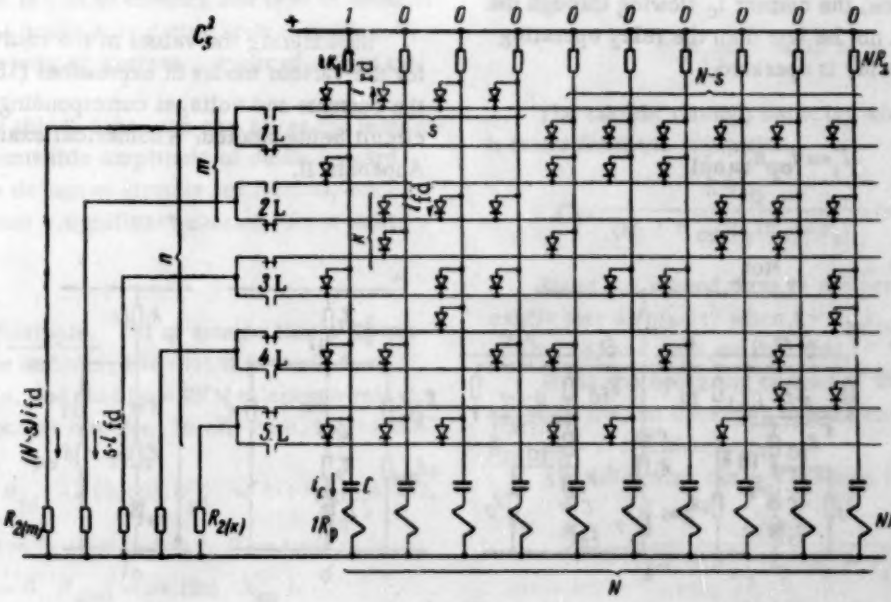

Fig. 3.

ficiently high, since the transient and steady-state current are basically determined by R_{rd}.

Therefore, all these circuits, for practical purposes, can be replaced by an equivalent resistance expressed in terms of the back resistance of a diode.

The second type of circuit contains the resistance R_g, which, as in the case examined earlier, determines the operating mode of the relay, and may change from 0 to ∞.

The circuits mentioned are shown in Fig. 4, b, where

$$R_4 = \frac{R_{rd}}{(n-m)(N-s-1)+m(s-1)}.$$

The computational equivalent circuit is shown in Fig. 4, c, where R_{eq} will assume different values which are a function of the operating mode. When $R_g = \infty$, the equivalent resistance will equal

$$R'_{eq} = \frac{R_{rd}}{(n-m)(N-s-1)+m(s-1)} \qquad (10)$$

and when $R_g = 0$

$$R''_{eq} = \frac{R_{fd}}{m}. \qquad (11)$$

We will derive relations for the given circuit, assuming that all quantities entering them are constant. We can write the equation

$$u = (i_c + i_1)R_1 + i_c R_p + L_p \frac{di_c}{dt} + u_c \qquad (12)$$

from the computational equivalent circuit.

In this equation, the current i_c flowing through the given circuit must not be less than the relay operating current, when the relay is operated;

$$i'_c = i_{op} k_{rs(op)}$$

but must be less than the relay drop-out current for all the remaining cases:

$$i''_c = \frac{i_{do}}{k_{rs(do)}},$$

where $k_{rs(op)} > 1$, and $k_{rs(do)} > 1$.

Manipulating equation (12) further, we obtain a second-order differential equation

$$\frac{d^2u_c}{dt^2} + \frac{R_p R + R_1}{RL_p}\frac{du_c}{dt} + \frac{1}{L_p C}u_c = u, \qquad (13)$$

where $R = \left(\frac{R_1}{R_{eq}} + 1\right)$, L_p is the inductance of the relay coil; R_p, the relay resistance; R_1, the matrix input resistance; C, the capacitance of the isolating capacitors; and u, the applied voltage.

The roots of the characteristic equation corresponding to equation (13) are

$$\alpha_{1,2} = -\frac{R_p + \frac{R_1}{R}}{2L_p} \pm \sqrt{\frac{R_p + \frac{R_1}{R}}{4L_p^2} - \frac{1}{L_p C}}. \qquad (14)$$

Integrating equation (13), we find the current through the relay and the voltage across the relay coils and capacitor:

$$u_c = \frac{uR_{eq}}{R_1 + R_{eq}}\left[1 - \frac{1}{\alpha_2 - \alpha_1}(\alpha_2 e^{\alpha_1 t} - \alpha_1 e^{\alpha_2 t})\right], \qquad (15)$$

$$i = -\frac{uR_{eq}}{(R_1 + R_{eq})L_p(\alpha_2 - \alpha_1)}(e^{\alpha_1 t} - e^{\alpha_2 t}), \qquad (16)$$

$$u_L = -\frac{uR_{eq}}{(R_1 + R_{eq})(\alpha_2 - \alpha_1)}(\alpha_1 e^{\alpha_1 t} - \alpha_2 e^{\alpha_2 t}). \qquad (17)$$

Substituting the values of the equivalent resistances for the various modes in expressions (15)–(17), we find the currents and voltages corresponding to them in the circuit being studied. A numerical example is given in Appendix II.

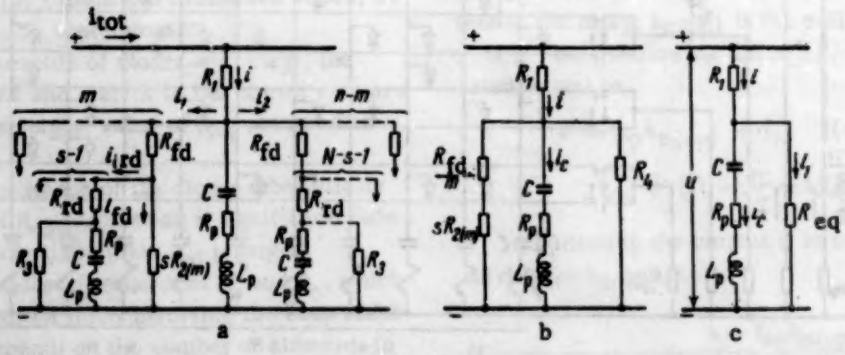

Fig. 4.

CONCLUSIONS

One can draw the following conclusions from the given analysis of the operation of the decoders mentioned above, and also from experimental work on them done by the author.

1. The choice of decoder matrix parameters (parameters of diodes, relays, etc.), and of the supply voltage for continuous and pulse powering of the latter, depends on the type of matrix ($N = C_n^m$), that is, the total number of output circuits, combinational elements, code elements, number of defensive circuits, and number of connections between the input and output circuits.

2. The operating mode of the decoder–a reliable selection of the output relay corresponding to the given code, and a defensive rejection if this code is distorted in any way–is basically determined by the matrix resistance R_2. The best results in the given two cases are obtained when this resistance changes from 0 to ∞.

3. The back resistance of the matrix diodes substantially affects the operation of the decoder output relays, and this effect increases as the total number of matrix output circuits grows.

A decrease in the diode back resistance or an increase in the total number of output circuits leads to a decrease in the relay operating reserve factor when a code is present, and in the drop-out reserve factor when no code is present, if the voltage is fixed. In a number of cases, this circumstance makes it necessary to increase the supply voltage.

4. The energy content of matrix decoders with relay output is relatively large, and is determined by the type of matrix and the output relay parameters, and also depends on the operating mode of the decoder. A large amount of power is required in the absence of code, both for continuous and for pulse-powering.

Therefore, it is best to connect this type of decoder to supplies of low power only during code reception, even if there are only an average number of output circuits (15)–(20).

Moreover, if this is done, one can lower the requirements on the permissible amplitude of diode forward current when the defensive circuits are formed, since the diodes can take a significant overload for a short time.

APPENDIX I

Numerical Example. Let us assume that a C_5^2 matrix is used in the decoder, and that it is made from type D7Zh diodes, and that type RKN telephone relays are used in the output circuits. In this case, the basic data will be

$$R_p = 1000 \text{ ohm}, R_{fd} = 10 \text{ ohm}, R_{rd} = 5 \cdot 10^5 \text{ ohm}, k = 2,$$

$$N = 10,\ n = 5,\ m = 2,\ s = 4,\ N - s - 1 = 5,$$

$$R_{2(k)} = 0,\ R_{2(m)} = \infty \text{ (for } R'_{eq}),$$

$$R_{2(k)} = R_{2(m)} = 0 \text{ (for } R''_{eq})$$

Computations give the following results:

$$R'_{eq} = \frac{R_p}{1 + 0.011 + 0.03} = \frac{R_p}{1.04} = 996 \text{ ohm},$$

$$R''_{eq} = \frac{R_p}{1 + 20.0 + 0.03} = \frac{R_p}{201.03} = 4.97 \text{ ohm}.$$

APPENDIX II

As an example, let us find the current flowing through a relay for the case computed in Appendix I, assuming that the relay coil inductance is $L_p = 10$ h, and the capacitance $C = 16$ μf; all the remaining quantities are the same as in the preceding example.

The operating and drop-out currents of the relay are, respectively, $i_{op} = 20$ ma, $i_{do} = 9$ ma, $R_1 = 500$ ohms.

When the relay is operated, we get

$$R'_{eq} = \frac{R_{rd}}{(n-m)(N-s-1)+m(s-1)} = \frac{500 \cdot 10^3}{3 \cdot 5 + 2 \cdot 3} = 24 \cdot 10^3 \text{ ohm}$$

When the relay is idle, we get

$$R''_{eq} = \frac{R_{fd}}{m} = \frac{10}{2} = 5 \text{ ohm}.$$

Let us find the resistance R for these two cases:

$$R' = \frac{R_1}{R'_{eq}} + 1 = 1.021 \text{ ohm},$$

$$R'' = \frac{R'}{R''_{eq}} + 1 = 101 \text{ ohm}.$$

The roots of the characteristic equation for the relay in the operated state are

$$\alpha'_1 = 0,\ \alpha'_2 = -150;$$

and for the relay idle–

$$\alpha''_1 = -34.6;\ \alpha''_2 = -66.$$

The current through the relay while it is operated is found from the expression

$$i' = \frac{uR'_{eq}}{(R_1 + R'_{eq})L_p(\alpha'_2 - \alpha'_1)}(e^{\alpha'_1 t} - e^{\alpha'_2 t}) =$$

Since the second term in the parenthesis is significantly less than unity when $t = t_{op} k_{rs(op)}$, it can be neglected, and then we find that $i \approx 40$ ma.

Since the operating current of the relay is equal to $i_{op} = 20$ ma, an operating current reserve factor, $k_{rs(op)} = 2$, is ensured.

The idle relay current is found from the formula

$$i'' = -\frac{uR''_{eq}}{(R_1 + R''_{eq})L_p(\alpha''_2 - \alpha''_1)}(e^{\alpha''_1 t} - e^{\alpha''_2 t}).$$

When $t = t_{do}$, we find that $i'' \approx 0.46$ ma, which is significantly less than the operating current.

ON THE INTERFERENCE RESISTANCE OF PULSE-TIME AND PULSE-WIDTH TELEMETERING SUBJECTED TO STRONG FLUCTUATION NOISE

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The possibility is discussed of raising the interference resistance of pulse-time and pulse-width telemetering by introducing a time selector into the receiver; the selector would eliminate the effect of short duration, but highly probable, errors formed by the action of fluctuation noise.

Video pulse errors are formed in the video channel of a receiver by the action of strong fluctuation noise. In general, these errors are false pulses and false spaces of different duration.*

The influence of strong fluctuation noise on the transmission of PWM and PTM telemetering is analyzed in reference [1]. It is shown that the average and mean-square errors are determined by the probabilities, P_s and P_p, of the unit errors of the space and pulse. The unit error is defined as the formation, by noise, of a false pulse or false space of duration $\Delta t = 1/W = 1/2f_c$, where $f_c = W/2$ is the cut-off frequency of the low-pass filter in the video channel, which is replaced by a band-pass filter if there is no secondary modulation before transmission.

It is assumed in [1] that the receiver is sensitive to errors of duration Δt.

Is this really necessary? It is true that an input filter with pass-band W creates the necessary conditions for passing pulses Δt wide. But the pass-band of the input filter is designed to be wide enough to pass working pulses with a sufficiently fast rise-time. If transmission stability requirements oblige one to pass pulses with a fast rise-time, this requirement does not extend to the width of those pulses.

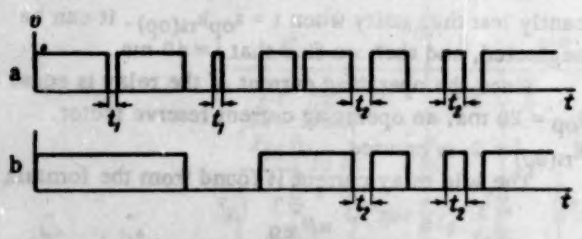

Fig. 1. Pulses in the video channel of a receiver. a) before the time selector; b) after the time selector ($t_1 < t < t_2$).

By taking special measures, such as introducing circuits in the video channel of the telemetering receiver which delay time by $\underline{t}$, and which make the receiver insensitive to short-duration errors, one can significantly increase the interference resistance of the receiver. This remark is intended to draw attention to this simple method, and to make the necessary additions to the analysis of interference resistance in reference [1], where it is assumed the main influence on the indications of the receiver is the extremely short, but highly probable, errors of unit duration, Δt.

Ideally, such a device, which can be called a time selector (filter), or duration selector, is triggered by the leading edges of the incoming pulses. Pulses are generated at the output which are time-shifted by $\underline{t}$, and which duplicate the input channel, providing it does not contain pulses or spaces shorter than $\underline{t}$. Pulses shorter than $\underline{t}$ do not trigger the device, and likewise, spaces shorter than $\underline{t}$ cannot restore it to the initial state. Thus, the time selector does not pass signals which are less in duration than some set value, $\underline{t}$ (Fig. 1).

This type of apparatus can be made in different ways. The simplest is a relay which is slow to operate and release, and which is actuated by constant amplitude pulses. In general, if there is double limiting in the video channel (Fig. 2), a low-pass filter can be used as a time selector (for instance, an integrating RC circuit), if the method of demodulation after this is such

* Telemetering subjected to strong noise is ordinarily accomplished by transmission in which the rise time of the working pulses is very short. The permissible width of the leading edges in this case is of the order of the indeterminateness permissible in measuring time intervals, and distortions of the leading edges by noise can be neglected.

that the "piling" of the leading edges of the working pulses in the filter does not lower the measurement accuracy. If this is not so, a more complex time selector must be used†.

As is mentioned in [1], the duration of the unit error is the correlation interval of the smooth noise at the filter output, and thus the formation of unit errors can be considered independent. If the probabilities of unit errors (that is, errors of duration Δt) in a space, P_s, and pulse, P_p, are known, then the probabilities of errors which have a duration equal to the delay time, t, are found in the following manner:

$$P(t)_s = P_s^k, \text{and } P(t)_p = P_p^k \qquad (1)$$

It is certain, from the standpoint of interference resistance, that the greater the delay,

$$t = k\Delta t, \qquad (2)$$

the better—that is, the less—the error due to noise.

However, the delay cannot be too great; it is limited by the following factors.

First of all, an increase in the delay causes an increase in the minimum values of the pulses and spaces, whose duration must exceed the delay introduced by the time selector. As a result, the useful time deviation which carries information is decreased somewhat, and the telemetering accuracy is reduced, or the rapidity of response is lowered as the cycle duration is correspondingly increased. But the magnitude of these factors is not great, and it is seen that in practical devices, they do not limit the delay.

The second most important factor limiting an increase in the delay is the instability of the device which introduces this delay.

The greater the delay, the more difficult it is to meet the stability requirements. Hence, it is evident that a relatively unstable, but small, delay introduces a large error in the indications of the telemetering receiver.

A telemetering receiver insensitive to short errors of duration Δt would be of great practical value. To a lesser degree, it is also desirable to eliminate the less probable errors of duration $2\Delta t$, etc. If the duration, Δt, is comparable to the rise time of the pulse, that is, to the telemetering accuracy under conditions of strong interference, it is not difficult, in practice, to ensure a sufficiently stable delay, $t = (2-3)\Delta t$, that is, a coefficient $k = 2-3$.

In order to evaluate the interference resistance of PWM and PTM telemetering apparatus, incorporating a time selector with delay t(2) in the receiver, the probabilities P_s and P_p in the design formulas derived in [1] must be replaced by P_s^{k+1} and P_p^{k+1} respectively.

Simplified expressions for the average (δ_{av}), and mean square (δ_{ms}) reduced errors are given in the following table:‡

Method of modulation	Without delay		With delay	
	δ_{av}	δ^2_{ms}	δ_{av}	δ^2_{ms}
PWM	$\frac{P_s}{2}$	$\frac{P_s}{2Nn}$	$\frac{P_s^{k+1}}{2}$	$\frac{P_s^{k+1}(k+1)}{2Nn}$
PTM	$\frac{nP_s}{6}$	$\frac{nP_s}{12N}$	$\frac{nP_s^{k+1}}{6(k+1)}$	$\frac{nP_s^{k+1}}{12N(k+1)}$

† In practice, the ideal time selector is based on a scaler circuit, triggered and blocked by the respective leading edges of pulses.

‡ We must keep in mind that the formulas presented in the table do not take into account possible error at the edges of the pulses and spaces. The probability of these errors is low, but the error component due to "edge" errors will increase as the delay is increased.

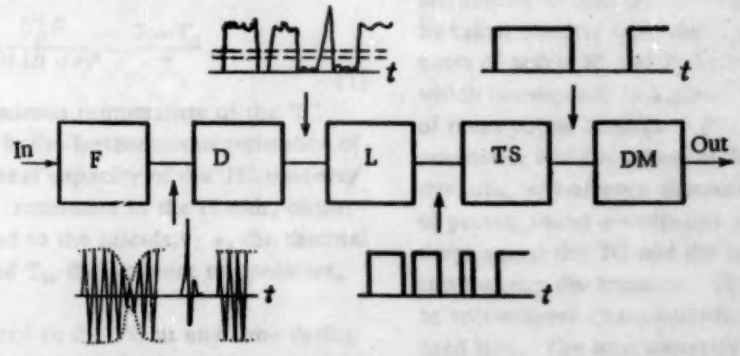

Fig. 2. Block-diagram of a telemetering receiver using PWM and a time selector. F-band-pass filter, D-amplitude detector, L-limiter, TS-time selector, DM-demodulator-this transforms pulse duration into proportional amplitudes of voltage or current.

SUMMARY

The introduction of duration selection in the video channel of a receiver significantly increases the interference resistance of PWM and PTM telemetering. Such a method is especially valuable in telemetering apparatus working under conditions of strong interference, since there is no strict relation between the durations of the pulse, and of the leading edge: the pass-band of the filter determines the rise time, but the duration of the shortest pulses can vary over wide limits.

The introduction of long delay, two to three times more than the duration of a unit error, Δt, is not difficult to realize, and can be used practically in all time-type telemetering apparatuses. In connection with this, it is best to examine the data of reference [1] on the comparative interference resistance characteristics of telemetering devices which use different modulation methods. Such a comparative analysis will be the subject of a special article.

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** See English translation.

AN APPROXIMATE CALCULATION OF THE TRANSIENTS IN A THERMISTOR CIRCUIT

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A form of the differential equation for heating a thermistor by the current flowing through it is proposed; with it, one can obtain all the variables necessary for calculating the transient directly from a graph of the volt-ampere characteristic.

A graphical method of computing the transients arising in a thermistor circuit (TC) excited by a step change in the parameters which characterize the operating mode of the circuit is presented in [1], and is further developed in [2, 3]. This method has a great disadvantage: the thermal time constant, τ, whose magnitude greatly depends on temperature, enters the calculation. Assuming τ to be constant results in significant error in calculating the transients, which are accompanied by a temperature change of hundreds of degrees in the working body of the TC. This disadvantage can be eliminated by a method suggested in [4], where a form of the differential equation for the heating of a TC is given in which the thermal time constant is eliminated from the calculation. However, the solution of the problem in this case remains quite complicated.

Still another form of the differential equation is suggested below; it permits one to curtail the amount of computation significantly, and makes the calculation quite graphic.

The following form of the heating differential equation is recommended in [1] for convenience in calculating the rate of temperature change in the working body of a TC at various temperatures:

$$\frac{dT}{dt} = \frac{U_p^2 R}{H(R+r)^2} - \frac{T-T_0}{\tau}, \tag{1}$$

where T is the instantaneous temperature of the TC working body, in °K; R, the instantaneous resistance of the TC, ohms; H, the heat capacity of the TC, watt-sec/deg; r, the linear resistance in the circuit, ohms; U_p, the voltage applied to the circuit, v; τ, the thermal time constant, sec; and T_0, the ambient temperature, °K.

The power liberated in the TC at any time during the transient can be found from the formula,

$$\frac{U_p^2}{(R+r)^2} R = U'I',$$

where U' is the instantaneous voltage drop across the TC, and I' is the instantaneous current.

The time constant is defined by the expression $\tau = H/b$, where b is the dissipation coefficient. Taking this into account, we can rewrite expression (1) as

$$\frac{dT}{dt} = \frac{U'I' - b(T-T_0)}{H}$$

The product of the dissipation coefficient b and the temperature of overheating $(T-T_0)$, which appears in the above expression, is equal to the power dissipated by the TC into the surrounding medium under steady-state conditions at a temperature of the working body T, i.e.,

$$b(T-T_0) = UI,$$

where U is the voltage drop across the TC in the steady-state, and I is the steady-state current through the TC.

Consequently,

$$\frac{dT}{dt} = \frac{U'I' - UI}{H}. \tag{2}$$

Turning to the volt-ampere characteristic on which the load line (U_p-I) is drawn (Fig. 1, a), it is easy to see that all the quantities dependent on temperature which are needed to find the TC temperature change rate can be taken directly from the figure; these are the coordinates of points K' and K, lying on the resistance line which corresponds to a given TC temperature, T. One of these points belongs to the dynamic volt-ampere characteristic, and the other, to the static. We will call the dynamic volt-ampere characteristic the geometric locus of points, whose coordinates are equal to the voltage drops across the TC and the current through it at any time during the transient. In the given case, the dynamic volt-ampere characteristic is the segment 1'-2, of the load line. The heat capacity can be found by the method suggested in [4].

Let us illustrate the technique of computing a transient by a concrete example. It is necessary to plot a

curve of the current increase in a circuit consisting of a type KMT-11 TC and a linear resistance, r, connected in series. The transient arises when r is changed from r_1 = 180 kohms to r_2 = 12.65 kohms in one step. The TC is immersed in transformer oil at 20°C. The voltage, U_p, applied to the circuit is 220 v. The temperature characteristic of the TC is described by the expression $R = 0.281\, e^{\frac{4600}{T}}$, and the heat capacity, H = 2.78 mw-sec/deg. The volt-ampere characteristic of the TC, with resistance lines corresponding to eleven different temperatures, is shown in Fig. 1, a.

We begin the calculation by determining the steady-state modes of the TC before the start and after the completion of the transient. Load lines, U_p–Ir_1, and U_p–Ir_2, must be drawn for this purpose. Point 1 characterizes the state of the TC before the transient starts (R=860 kohms, T=309°K). After the transient is completed, the state characterized by point 2 is established (R=4.4 kohms, T=474°K).

The temperature change rates of the TC at different temperatures occuring during the transient are calculated from formula (2). For example, when T=408° K, the point K' on the dynamic volt-ampere characteristic has the coordinates: U'=134 v, I'=6.2 ma, and point K on the static characteristic has coordinates U=88 v, I=4 ma, and consequently,

$$\frac{dT}{dt} = \frac{134 \cdot 6.2 - 88 \cdot 4}{2 \cdot 78} = 171 \text{ deg/sec}.$$

The rates found by this method are plotted on the graph dT/dt = f(T), for the corresponding temperatures (Fig. 1, b).

Then a curve of the temperature change as a function of time is plotted (Fig. 1, c). To do this, the entire temperature change interval is divided into a series of

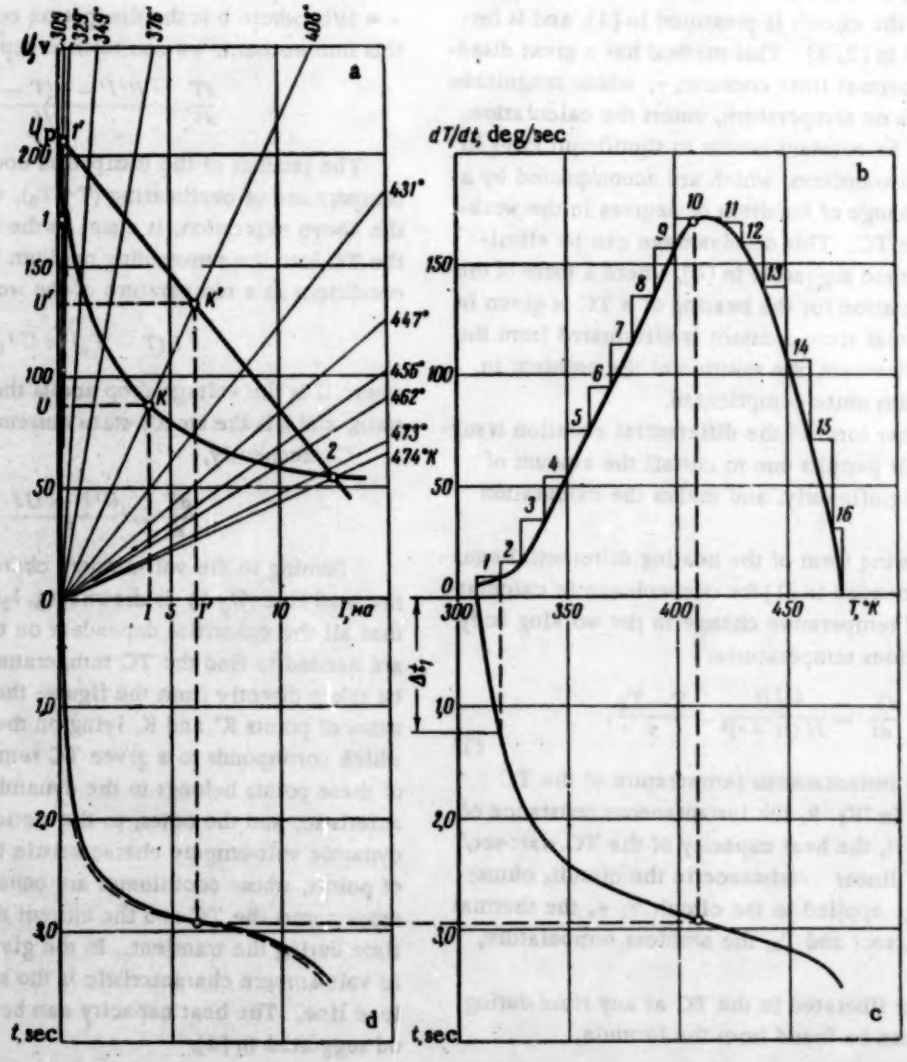

Fig. 1. An example of the calculation of a transient. a) initial volt-ampere characteristic; b) curve of the dependence of the TC working body temperature change rate on temperature; c) temperature change curve; d) current change curve.

segments, ΔT (16 parts, in the given example) within which the temperature is assumed constant, and the corresponding time intervals are computed. Thus, when the rate is $dT/dt = 9$ deg/sec, a temperature increment $\Delta T = 11$ deg (interval 1) occurs during the time $\Delta t = 11:9 = 1.22$ sec. By adding all the time intervals Δt, we construct the curve $T = f(t)$.

The construction of the curve $I = f(t)$ is clear from the drawing (Fig. 1, d).

The curve of current change obtained by calculation almost coincides with the experimental results (the dashed curve in Fig. 1, d). The experimental curve, like the initial data for the example under consideration, was taken from [5].

Transients arising as a result of a step change in the supply voltage or a change in heat-transfer conditions are calculated analogously. The volt-ampere characteristic needed for the calculation can be found analytically [6].

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THE SEVENTH SCIENTIFIC-TECHNICAL CONFERENCE OF YOUNG SCHOLARS OF THE INSTITUTE OF AUTOMATION AND REMOTE CONTROL OF THE ACADEMY OF SCIENCES OF THE USSR

On March 14-16, 1960, was held the traditional (seventh) Scientific-Technical Conference of Young Scholars of the Institute of Automation and Remote Control (IARC) of the Academy of Sciences of the USSR on Problems of Automatic Control. The conference discussed the work of young scholars which was completed in 1959.

More than 400 persons participated in the work of the conference, including about 200 representatives of various organizations in Moscow and the Moscow area (scientific research institutes, institutes of higher learning, planning and design organizations, etc.).

Seventy-five reports and communications were heard, encompassing a broad range of problems of automatic control. The materials presented evoked a lively exchange of opinions.

In opening the conference the director of the Institute of Automation and Remote Control, Academician V. A. Trapeznikov, wished the young scholars success in their work and charged them to link their scientific work more closely with the practical, with the concrete needs of the national economy. At the plenary session Professor M. A. Aizerman, D. T. S., presented a report on "The scientific problem of the theory of finite automata."

At the final plenary session E. V. Voloshina and E. V. Shtil'man presented a report on "Modeling the processes of instruction." In the report they reviewed the characteristic features peculiar to the processes of instruction of animals and man, surveyed a number of automatic devices which fulfill certain functions of the human brain, and showed analogous features between certain processes in automatic systems and processes of instruction of people and animals. Between the plenary sessions, the work of the conference was done in sections:

I. Automatic control, with subsections on the theory of automatic control and systems of automatic control
II. Automatic inspection
III. Computers
IV. Elements and devices of automation and remote control
V. Statistical methods in automation
VI. Theory of relay circuits and finite automata
VII. Automated electric drive

I. Automatic Control Section

A. Subsection on the theory of automatic control

A report by V. N. Novosel'tsev was devoted to finding an expression for optional control in secondary relay-impulse systems for the case of purely relay control and for the case of relay control in the presence of a zone of insensitivity.

I. S. Morosanov discussed in his report the effect of fluctuations on relay extremal systems under auto-oscillating conditions. On the basis of evaluation the principles of harmonic and statistical linearization were assumed. The author showed that the methods of calculating statistical transfer coefficients in the form proposed by I. E. Kazakov are inapplicable in this case. But a linearization equivalent in probable meaning can be obtained in reviewing the problems of parametric effect of fluctuations. The final solution for the case of linear members of an extreme object of any order was given in graphic form. As a result, on the basis of published statistical characteristics of the fluctuations, the mean value of the frequency and amplitude of the auto-oscillations is found for a given system of extreme control.

The joint report of V. G. Gradetskii and Yu. I. Ostrovskii entitled "The operation of a system of extreme control (SEC) with memory of the extreme under noise conditions" was devoted to the problem of raising the noiseproof features of systems. The value used in the report as an index of the noiseproof feature characterizes the number of false switchings of the executive organ in a unit of time.

The task amounts to a determination of those values of the parameters of tuning of the regulator and noise filter which, at a given rate of scanning and a given loss in scanning, would provide the minimal amount of false switchings in a unit of time. This problem was solved approximately on account of its complexity. The formulas obtained have been verified by experiment.

A report by N. V. Grishko presented the results of determination of the optimal characteristics of a single extreme system during occasional functioning. The system reviewed is a natural generalization of several very noiseproof and high-grade extreme systems. The optimal characteristics are determined by varied methods by minimization of the analytical expression obtained for the mean square error through the given characteristics of the object being controlled and the characteristics produced by the extreme system.

T. G. Babunashvili devoted his report to a study of the triggering process of a regulating system containing a servomotor with a nonlinear velocity characteristic and

fixed feedback. The task consisted of investigating systems with three differential first-order equations, with coefficients variable in time and one nonlinearity from the point of view of dissipativity, largely in accordance with the thinking of Kh. Masser. The author of the report determined the maximal deviations and the deviations which are determined at the end of the triggering process.

In the report of A. I. Moroz entitled "Qualitative investigation of differential equations obtained in the solution of the problem of synthesis," a method of designing regulating systems was explained. It was suggested by A. M. Letov and consisted of the fact that the equation of the regulator was defined by the task of the first integral of the system of differential equations of the object and the unknown regulator. In an occasional case the advantages and deficiencies of this method have been noted, and certain unacceptable features of the systems obtained have been disclosed. The report proposed a method of eliminating the deficiencies discovered.

A report by M. M. Simkin was devoted to the determination of periodic conditions in nonlinear impulse systems. The indicated problem is linked with a review of nonautonomous systems.

The author proposed an approximate frequency method of determining simple and complex periodic conditions in such systems.

In a report of R. P. Parsheva the problem of stability in the Lagrangian sense of transient conditions in a pentametric SAR with a single nonlinearity was discussed.

V. S. Kislyakov, in a report on "The prolonged stability of an aircraft with an automatic pilot possessing retardation," investigated the stability of short-period motion of an airplane with an automatic pilot without feedback. Disturbed movement of the system is described by third-order differential equations with constant coefficients and retardation in the characteristic of the steering gear. To determine the region of asymptotic stability as a function of retardation and adjustments of the automatic pilot, on the assumption that the characteristic of the steering gear has weak nonlinearity, the method of asymptotic approximation of N. M. Krylov and N. N. Bogolyubov was used; this was also utilized in studying the linear problem. The effect of retardation on stability of movement was explained.

In the report of B. E. Chuprun on "The formulation of a law of control of linear stable objects assuring minimal displacement of the regulating organ," a method was studied of controlling a constant-rate servomotor, in which the regulating organ deviates only to the amount needed for compensation for the displacement. It is assumed here that the form of displacement is known, but its level is not. In like manner a study was made of a case of a linear stable object of arbitrary order without and with retardation.

B. Subsection on systems of automatic control

B. B. Buyanov discussed in his report the application of the theory of systems optimal for high speed to the problem of controlling the flying shears of a rolling mill.

B. G. Volik reported on an automatic optimizer with two channels and two limitings intended to make scannings of the extreme directly on the object which has great inertness. The problem was studied with regard to the automation of chemical objects. I. N. Bocharov reported on an original instrument for photographing the distribution curve of a random value.

E. A. Rateeva reported on a three-channel optimizer for chemical production, calculated for 2-3 variables and two limiters.

In the report of V. G. Sholokhov, experimental proof was given for convergence in the process of adjusting a noise generator, by the filter-forming method, for the case of two filters, and the dependence of the time of the transitional period on the method of triggering was determined.

The report of K. B. Norkin was devoted to the problem of automatic adjustment of a transmitter's output cascade with use of a system of automatic triggering, and contained an explanation of the method of optimal selection of adjustments of the optimizator during operation with an inert object under noise conditions.

In a report of V. N. Shadrin a system of program control with frequency separation of the channels was described.

Two reports made by V. V. Karibskii and A. P. Evseeva contained data on a universal interpolator for a system of digital program control and on the automatic selection of interpolation segments for a machine with a linear interpolator.

A report by M. L. Taits described an experimental-industrial model of an instrument for automatic program installation. Reports of B. N. Andreichikov dealt with problems of the dynamic precision of machines with program control and the effects of friction and slack on the error during reverse operation.

The report of M. M. Khasanov was devoted to an analysis of the dynamic characteristics of air-conditioning equipment.

II. Automatic Inspection Section

M. V. Rybashov and I. M. Ponasenko devoted their report to a study of several circuits for the dynamic compensation of counters. The conclusions obtained were confirmed by experimental investigations on models.

V. S. Likhoninskii, in his communication "A capacitance-measuring device for digital systems of program control of productive processes," examined the operating principles of capacitance-measuring devices of the linear and rotary types for the measurement of shifts.

M. A. Prusov reported on the principles of building and designing an instrument for measuring the temperature of rotating parts. The instrument is based on utili-

zation of turbulent currents and has a number of advantages in comparison with analogous instruments based on other principles.

The report of Yu. V. Gushchin was devoted to study of the possibility of using semiconductor crystal layers on a sulfur-cadmium base for the indication of radioactive radiations.

A. A. Kalmakov discussed in his report the possibility of using radioactive isotopes and the methods of nuclear spectroscopy for automatic control of the content of metals in alloys and products of the enrichments of ores of nonferrous metals.

V. A. Viktorov reviewed the effect of the high harmonics of a coaxial vibrator on the operation of an extremal endovibratory level indicator.

III. Computer Section

In a report by A. G. Butkovskii, "The modeling of certain objects with distributed parameters," the theoretical principles and a structural scheme are given for an electronic model of straightaway heat-treatment furnaces operating in an automatic-control system.

A. V. Shileiko, in his report on "Method of synthesis of an optimal structure of a digital model," examined problems linked with the selection of criteria of quality and method of synthesis of an optimal structure of a digital model, for which criterions of quality he assumes extreme values.

In a report of R. N. Chernyshev, "A solver amplifier with powerful output cascade," the results were given of the development of a solver amplifier, completed according to an autocompensating scheme. The amplifier has a contactless modulator on silicon diodes and a powerful output cascade.

In a report of B. A. Pereverzev entitled "Combined electromechanical assembly of an electronic modeling device" there were descriptions of the circuits, design, and basic technical data of an electromechanical unit which can operate under the conditions of a multiplier-divider, a functional converter and a coefficient buncher.

F. B. Gul'ko, in a review report entitled "High-speed electronic multiplying devices," furnished data on new electronic multiplying devices with an elevated transmission band, their characteristics, and the factors limiting high speed.

Zh. A. Novosel'tseva, in a report entitled "Controlled retardation assembly," gave the results of the development of a device for the modeling of retardation, the time of which changes in accordance with the controlling signal. The device was made for memory condensers which are switched with the help of step-by-step switches.

In a report of K. B. Norkin, "On a single method of automatic triggering of the extreme of a function of some variables," a method is given for discovering the extreme of a function which has lumps and saddles which do not require calculation of the gradient of the function.

V. A. Yakovlev, in a communication entitled "A discrete electrical differentiator," described the operating principle and design of a device for the differentiation of slowly changing voltages, which permits the calculation of the difference in voltages (from 20 mv to 2 v) at a discrete moment of time with an error not exceeding 1% of the maximum value of the scale.

V. A. Brik, in a report entitled "A digital computer for the composition of a program for the working of parts on a milling machine," discussed a device for preparing a program for the working of parts whose profiles consisted of rectilinear segments and circles. On the basis of the coordinates of the ends of the segments and the centers of the circles, the devices give the values of the linear increments of the coordinates in a binary code.

IV. Section on Elements and Devices of Automation and Remote Control

In a report of E. A. Andreeva a method was given for calculating the expense and power characteristics of a "jet-choke" element for viscous compressible and incompressible liquids.

L. A. Tenenbaum, in introducing simplified assumptions and hypotheses, derived expressions for the expense and power characteristics of a "jet-choke" element during the irregular laminar flow of a viscous incompressible liquid.

T. K. Efremova discussed pneumatic relay assemblies with temporal dependencies (guard relay, impulse starter, and impulse generator). In her report there was given a timer circuit built with these assemblies.

In a report of V. S. Matorina entitled "Magnetic output amplifiers for alternating current electromagnets," the operation of magnetic amplifiers working from the signals of magnetic logical elements was examined. Two methods of calculation for loads with high and low goodness were offered.

In M. A. Boyarchenkov's report "Standby magnetic amplifiers for direct current with increased efficiency," he analyzed the conditions for obtaining maximal power under load. New circuits of three-phase standby amplifiers and full-wave transformerless standby amplifiers were offered.

In a second report M. A. Boyarchenkov discussed the operation of a magnetic amplifier for counter emf.

N. L. Prokhorov analyzed in his report existing memory circuits for magnetic logical elements from the point of view of continuity (the memory signal during exclusion and following inclusion of the power supply current). A new continuous memory circuit was suggested.

A. L. Rozovskii, in his report entitled "A contactless code-impulse system of telemeasurement," explained the principle of construction and the solving circuits of a system built of semiconductor and magnetic elements.

In a report entitled "Contactless program-calculating device for the automation of instruments for a line-

casting machine," N. V. Silaev described the circuit of such a device, based on the use of magnetic and semiconductor elements.

V. S. Serzhers reported on the possibility of constructing, from semiconductor elements, circuits of a proportional amplifier, a derivative unit, and an integrating unit for regulators for general use in industry. A typical designed assembly of units was worked out, using a printing arrangement.

B. B. Gogodevskii discussed transitional processes in electromagnetic mechanisms and the vibration of contacts. A method was given for estimating the error in calculating the dynamic characteristics of electromagnetic mechanisms.

V. Section on Statistical Methods in Automation

In a communication of L. P. Sysoev a solution was given for the problem of estimating the parameters and of detecting signals which depend nonlinearly on random parameters assumed for the background of gauss-correlated noise.

M. Yu. Gadzhiev discussed the games problem arising in the case of detection of a useful signal mixed with noise during independent change of the carrier frequency. During definite assumed interference relative to the class, formulas have been obtained for optimal composite strategy of opponents and evaluation of the game.

In a report of I. I. Palshev equipment with continuous-discrete functioning for the resolution of random functions in a canonical series was examined. Experimental results were given of the practical construction of canonical series for a definite class of random functions.

E. L. Nappel'baum communicated about the form of the optical operator for the discovery of a suitable signal on a background of normal noises with random distribution of dispersions.

The report of A. I. Teiman was devoted to problems linked with the distribution of selections of random functions. A limiting form of distribution selections was derived, and a new method offered for the evaluation of the distribution of selections of random functions for a given level.

In the communication of E. S. Kochetkov there was an examination of the problem of the construction of theoretically and practically optimal linear integral immiscible evaluations of the mathematical expectation of the correlational function of stationary random processes with continuous time. A comparison was made of the theoretically and practically optimal evaluations.

T. I. Tovstukha discussed the effect of random noise on the operation of systems of extreme control of step-by-step and sloped types. Working parameters of the system are selected from evaluations of the values of mathematical expectation and dispersions on the yield of the object of control.

In the report of V. M. Baikovskii the capacity of a channel with different-discrete modulation was determined during the absence of noise. It was shown that potentially this method of transmission has greater capacity than delta-modulation and impulse-code modulation. The connection of the rate of transmission of information during a different-discrete modulation, according to the channel with the fluctuating noises, was determined from the statistics of the transferable information.

V. M. Pomazan communicated about a theoretical and experimental investigation of time systems of telemeasurement with various cycle periods and different methods of indication (with memory or integrating output devices).

VI. Section on the Theory of Relay Circuits and Finite Automata

In the report of V. D. Kazakov entitled "The shape of minimal forms of symmetrical Boolean functions of an arbitrary number of variables," it was demonstrated that the minimal sp- and ps- forms of symmetrical functions have a completely determined form and on the basis of this an account is given of a method of direct determination of the minimal forms of arbitrary symmetrical functions of n variables. Approximated estimates were given of the maximal number of minimal forms there are in an arbitrary symmetrical function of n variables.

In a report of V. P. Didenko there was an explanation of a digital method of minimization of Boolean functions with calculation of unutilizable states. The method permits rapid obtaining of partial minimal forms and can be distributed on Boolean functions with a large enough number of independent variables. An examination was also made of the method of constructing bridge structures of relay devices on the basis of partial minimal forms.

V. V. Vorsheva at the beginning of her report gave a short review of existing work on the study of circuits with real contacts. By real contacts are understood relay contacts in which there can be arbitrary changes of sequence of the interdependence of normally open and normally closed contacts of one relay or another. In the report there are an analysis and a synthesis of the circuits operating on real contacts. A method was explained for obtaining a partial minimal form giving the circuit, when realized, contacts free of competition.

T. M. Aleksandridi devoted his report to problems of the synthesis of switching circuits on the basis of two logical operations, the Schaeffer prime and the function which is double it. In the report it was proposed to realize the Schaeffer function and the function which is double it in elements operating on the principle of switching the current with transistors of type p-n-p and n-p-n respectively.

Examples were given of the construction of logical circuits made up of the elements examined.

In the report of L. A. Gusev, entitled "On minimization of the structure of finite automata," a review was

given of the basic concepts of the theory of finite automata and a study made of the equivalent conversions of finite automata and the problem linked to them of minimizing the number of states of the finite automaton.

The report of O. P. Kuznetsov was devoted to a description of logical networks in which the delay times for the various elements are unequal. The author showed that the relationships determining the behavior of such networks are more complex than the known relationships occurring in each automaton. In particular, there exist autonomous logical networks, whose sequences of states are periodic.

In the report of V. D. Kazakov and V. V. Naumchenko entitled "On the realization of Boolean functions of n variables on contactless logical switches by the method of redetermination," a new approach to the minimization of relay structures was explained, consisting of the fact that a given Boolean function leads at the expense of unutilized states to an ordered (in particular, symmetrical) form, with the help of simple matrix conversions, easily mechanized. The realization of symmetrical functions can be done on the contactless models of neurons of Calberston and Makkalok-Pitts. Such a realization permitted a sharp reduction in the number of necessary elements in the circuit.

The report of A. D. Talantsev, entitled "On the use of logical-algebraic operators of transition to the analysis and synthesis of finite automata of a special type," was devoted to an examination of the logical means of synthesis and analysis of potential-impulse circuits operating under asynchronous conditions (on a nonqualitative time scale). It was shown that the procedure of integrating potential-impulse forms permits in some cases a reduction in the number of necessary logical elements by one-half or two-thirds.

VII. Section on Automated Electrical Drive

O. I. Khasaev, in his report entitled "The operation of an induction motor from a frequency converter with semiconductor triodes," presented an analytical and experimental investigation of fixed and transitional processes of an electrical impulse converter in the angle of rotation of an impulse operation with a three-stator step-by-step motor. Analytic expressions were found for the basic parameters of the converter (the angles of overlap and lead) and an analysis was given of some conditions of its operation.

In the report of V. D. Vershinin entitled "The use of the principle of invariantness in the stabilization of the speed of a direct current motor," there was an explanation of the conditions of complete invariantness during action on the voltage of a signal armature upon the current of the motor, and the possibility of it realization was shown. It was shown that the fulfillment of the conditions of complete invariantness in the actual circuit, without fail, brings upon itself instability of the system. Compensation during complete invariantness brings the system to astatism of an order equal to the number of compensated components (basic and productive) of the compelled error.

In the report of O. A. Kossov, entitled "The direct current drive with impulse semiconductor rectifiers," the economy of impulse regulation of the speed of a direct-current motor was proved in the presence of a diode shunting armature circuit, an analysis was given of the statistical characteristics of the broken system, and the effect of feedback was determined. The author presented experimental data on the test of a model of the drive with a 0.76 kw motor.

In the report of Chao Chou-lun entitled "Optimal control of cylindrical flying pruners with eccentrics," there was an examination of problems in improving the quality of SAR during introduction of feedback; criteria of the stability of a broken system were given, and the conditions for formation of autooscillations were found.

In the report of A. R. Dzhelyalov, entitled "A synchronous motor with longitudinal-transverse excitation as the object of automatic regulation," the statics of regulation during control of excitation on both axes were examined in connection with the problem of slewing of the rotor during mutual synphasing of electrical drives with a reciprocating motor. The author concluded that the control of motors is the object of automatic control, described the principles of construction of systems of automatic turning of the motor's rotor, analyzed the conditions of rotation, and also gave a qualitative characterization of those conditions.

M. Yu. Gadzhiev, F. B. Gul'ko,
A. R. Dzhelyalov, E. E. Dudnikov,
V. D. Kazakov, I. A. Litovchenko,
K. B. Norkin, and N. L. Prokhorov

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